P2 Chapter 7 :: Trigonometry And Modelling

Chapter Overview

This chapter is very similar to the trigonometry chapters in Year 1. The only difference is that new trig functions: sec, $cosec$ and cot, are introduced.

1a:: Addition Formulae

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

1b:: Double Angle Formulae

"Solve, for $0 \leq x < 2\pi$, the equation

$$2\tan 2y \tan y = 3$$

giving your solutions to 3sf."

3:: Simplifying $a\cos x \pm b \sin x$

"Find the maximum value of $2\sin x + \cos x$ and the value of x for which this maximum occurs."

4:: Modelling

"The sea depth of the tide at a beach can be modelled by $x = Rsin\left(\frac{2\pi t}{5} + \alpha\right)$, where t is the hours after midnight..."

Topics	What students need to learn:		
	Content		Guidance
5 **Trigonometry** *continued*	5.5	**Understand and use** $\tan\theta = \frac{\sin\theta}{\cos\theta}$ **Understand and use** $\sin^2\theta + \cos^2\theta = 1$ $\sec^2\theta = 1 + \tan^2\theta$ and $\text{cosec}^2\theta = 1 + \cot^2\theta$	**These identities may be used to solve trigonometric equations** and angles may be in degrees or radians. **They may also be used to prove further identities.**
	5.6	Understand and use double angle formulae; use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$, and $\tan(A \pm B)$, understand geometrical proofs of these formulae. Understand and use expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms of $r\cos(\theta \pm \alpha)$ or $r\sin(\theta \pm \alpha)$	To include application to half angles. Knowledge of the $\tan(\frac{1}{2}\theta)$ formulae will *not* be required. Students should be able to solve equations such as $a\cos\theta + b\sin\theta = c$ in a given interval.
	5.7	**Solve simple trigonometric equations in a given interval, including quadratic equations in sin, cos and tan and equations involving multiples of the unknown angle.**	**Students should be able to solve equations such as** $\sin(x + 70°) = 0.5$ **for** $0 < x < 360°$, $3 + 5\cos 2x = 1$ **for** $-180° < x < 180°$ $6\cos^2 x + \sin x - 5 = 0,\ 0 \leqslant x < 360°$ These may be in degrees or radians and this will be specified in the question.
	5.8	Construct proofs involving trigonometric functions and identities.	Students need to prove identities such as $\cos x \cos 2x + \sin x \sin 2x \equiv \cos x$.
	5.9	Use trigonometric functions to solve problems in context, including problems involving vectors, kinematics and forces.	Problems could involve (for example) wave motion, the height of a point on a vertical circular wheel, or the hours of sunlight throughout the year. Angles may be measured in degrees or in radians.

Addition Formulae

Addition Formulae allow us to deal with a **sum or difference of angles**.

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$
$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$
$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Do I need to memorise these?
They're all technically in the formula booklet, but you REALLY want to eventually memorise these (particularly the sin and cos ones).

How to memorise:

First notice that for all of these the first thing on the RHS is the same as the first thing on the LHS!

- For sin, the operator in the middle is the same as on the LHS.
- For cos, it's the opposite.
- For tan, it's the same in the numerator, opposite in the denominator.

- For sin, we mix sin and cos.
- For cos, we keep the cos's and sin's together.

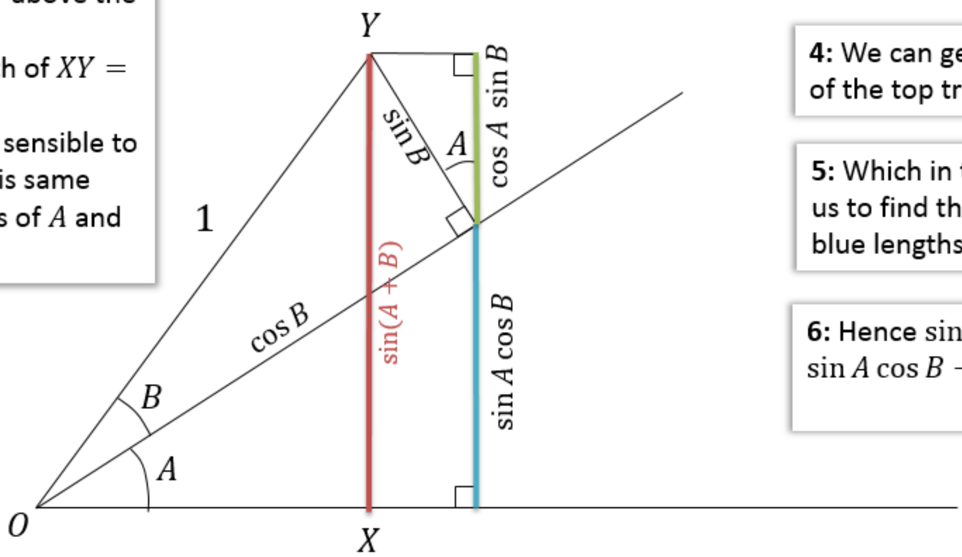
Proof of $\sin(A + B) \equiv \sin A \cos B + \cos A \sin B$

(Not needed for exam)

1: Suppose we had a line of length 1 projected an angle of $A + B$ above the horizontal.
Then the length of $XY = \sin(A + B)$
It would seem sensible to try and find this same length in terms of A and B individually.

2: We can achieve this by forming two right-angled triangles.

3: Then we're looking for the combined length of these two lines.



4: We can get the lengths of the top triangle...

5: Which in turn allows us to find the green and blue lengths.

6: Hence $\sin(A + B) = \sin A \cos B + \cos A \sin B$

□

Proof of other identities

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(a) Starting from the formulae for $\sin(A+B)$ and $\cos(A+B)$, prove that

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

(4)

Examples

Q Given that $2\sin(x + y) = 3\cos(x - y)$ express $\tan x$ in terms of $\tan y$.

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