7A Additional Formulae Identities

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

1. Use $cos(\alpha + \beta) = cos\alpha cos\beta - sin\alpha sin\beta$ to show that:

$$cos(A - B) = cosAcosB + sinAsinB$$

2. Use $sin(\alpha + \beta) = sin\alpha cos\beta + cos\alpha sin\beta$ and $cos(\alpha + \beta) = cos\alpha cos\beta - sin\alpha sin\beta$
   To show that

$$tan(A + B) = \frac{tanA + tanB}{1 - tanAtanB}$$

3. Prove that

$$\frac{cosA}{sinB} - \frac{sinA}{cosB} \equiv \frac{cos(A+B)}{sinBcosB}$$

4. Given that:

$2\,sin(x+y) = 3cos(x-y)$

Express $tanx$ in terms of $tany$

7B Applying the Addition Formulae

1. Given that:

$$\text{SinA} = -\frac{3}{5} \quad 180^{\circ} < A < 270^{\circ}$$

$$\text{CosB} = -\frac{12}{13} \quad \text{B} = \text{Obtuse}$$

Find the value of:

Tan(A+B)

7C Double Angles Formulae

$$\sin 2A \equiv 2\sin A\cos A$$

Key points:

If you see sin and cos multiplied together and both have the same angle -> this can be reduced using sin double angle formulae to a single trig function

Examples:

a) $\sin x\cos x =$

b) $6\cos x\sin x =$

c) $5\sin 3x\cos 3x =$

$$Cos2A \equiv Cos^2 A - Sin^2 A$$

Key points:

If you see $Sin^2 A$ or $Cos^2 A$ this can be reduced using the cos double angle formulae to a single trig function

Examples:

a) $Sin^2 x$ =

b) $4Cos^2 x$ =

c) $3Sin^2 4x$ =

$$Tan\ 2A \equiv \frac{2TanA}{1-Tan^2A}$$

1. Use the double angle formulae to write the following expression as a single trigonometric ratio:

   a) $Cos^2 50 - Sin^2 50$

   b) $\frac{2tan\frac{\pi}{6}}{1-tan^2\frac{\pi}{6}}$

   c) $\frac{4sin70}{sec70}$

2. Given that $x = 3sin\theta$ and $y = 3 - 4cos2\theta$, eliminate θ and express y in terms of x.

3. Given that:

$$cosx = \frac{3}{4}, \qquad 180^{\circ} < x < 360^{\circ}$$

Find the exact value of:

a) $sin2x$

b) $tan2x$

7F Trig Identities with Double Angles formulae

1. Show that:

$$2sin\frac{\theta}{2}cos\frac{\theta}{2}cos\theta \equiv \frac{1}{2}sin2\theta$$

2. Show that:

$$1 + cos4\theta \equiv 2cos^2 2\theta$$

3. Prove the Identity:

$$tan2\theta \equiv \frac{2}{cot\theta - tan\theta}$$

4. Prove the Identity:

$$\sqrt{3}cos4\theta + sin4\theta \equiv 2cos\left(4\theta - \frac{\pi}{6}\right)$$

7D Solving Equations

1. Solve $4cos(\theta - 30) = 8\sqrt{2}sin\theta$ in the range $0 \leq \theta \leq 360$. Give answers to 1dp.

2. Solve $3cos2x - cosx + 2 = 0$ in the range $0 \leq x \leq 360$. Give answers to 1dp where appropriate.

3. Solve $2tan2ytany = 3$ in the range $0 \leq y \leq 2\pi$. Give answers to 2dp.

4.

a) By expanding $sin(2A + A)$, show that:

$$sin3A \equiv 3sinA - 4sin^3A$$

b) Hence, or otherwise, solve:

$16sin^3\theta - 12sin\theta - 2\sqrt{3} = 0$, in the range $0 < \theta < 2\pi$

7E $\boldsymbol{RCos(x \pm \alpha)}$ & $\boldsymbol{RSin(x \pm \alpha)}$

1. Show that:

$$3sinx + 4cosx$$

   Can be expressed in the form:

$$Rsin(x + \alpha)$$

$$R > 0$$

$$0° < \alpha < 90°$$

2. Show that you can express:

$$\sin x - \sqrt{3}\cos x$$

In the form:

$$R\sin(x - \alpha)$$

$$R > 0$$

$$0 < \alpha < \frac{\pi}{2}$$

Quick Shortcuts and patterns:

Key point: This skill helps us reduce two trigonometric functions with the same angle that are being summed together to one single trigonometric function.

Where could this skill be useful?

3.

a) Express:

$$2cos\theta + 5sin\theta$$

in the form:

$$Rcos(x - \alpha)$$

$$R > 0$$

$$0° < \alpha < 90°$$

b) Hence, solve $2cos\theta + 5sin\theta = 3$, $0° < \theta < 360°$

7G Modelling with Trigonometry

Expression	Maximum	(Smallest) θ at max
$20\sin\theta$		
$5 - 10\sin\theta$		
$3\cos(\theta + 20°)$		
$\dfrac{2}{10 + 3\sin(\theta - 30)}$		

1.
2. The cabin pressure, P, in pounds per square inch (psi) on an aeroplane at cruising altitude can be modelled by the equation:

   $P = 11.5 - 0.5sin(t - 2)$

   **note these formulae are often a result of reducing to Rcos(x+a)* form

   Where t is the time in hours since the cruising altitude was first reached, and all angles are measured in radians

a) State the minimum and maximum cabin pressure

b) Find the time after reaching cruising altitude that the cabin first reaches a maximum pressure

c) Calculate the cabin pressure after 5 hours at a cruising altitude

d) Find all the times during the first 8 hours of cruising that the cabin pressure would be exactly 11.3 psi.