P2 Chapter 6 :: Trigonometry

Chapter Overview

This chapter is very similar to the trigonometry chapters in Year 1. The only difference is that new trig functions: $sec, cosec$ and cot, are introduced.

1:: Understanding $sec, cosec, tan$ and draw their graphs.

"Draw a graph of $y = cosec\ x$ for $0 \leq x < 2\pi$."

2:: 'Solvey' questions.

"Solve, for $0 \leq x < 2\pi$, the equation

$$2cosec^2 x + \cot x = 5$$

giving your solutions to 3sf."

3:: 'Provey' questions.

"Prove that

$$\sec x - \cos x \equiv \sin x \tan x$$

4:: Inverse trig functions and their domains/ranges.

"Show that, when θ is small,

$\sin 5\theta + \tan 2\theta - cos2\theta \approx 2\theta^2 + 7\theta - 1$."

Specification

5.4	Understand and use the definitions of secant, cosecant and cotangent and of arcsin, arccos and arctan; their relationships to sine, cosine and tangent; understanding of their graphs; their ranges and domains.	Angles measured in both degrees and radians.
5.5	**Understand and use** $\tan\theta = \frac{\sin\theta}{\cos\theta}$ **Understand and use** $\sin^2\theta + \cos^2\theta = 1$ $\sec^2\theta = 1 + \tan^2\theta$ and $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$	**These identities may be used to solve trigonometric equations** and angles may be in degrees or radians. **They may also be used to prove further identities.**
5.6	Understand and use double angle formulae; use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$, and $\tan(A \pm B)$, understand geometrical proofs of these formulae. Understand and use expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms of $r\cos(\theta \pm \alpha)$ or $r\sin(\theta \pm \alpha)$	To include application to half angles. Knowledge of the $\tan(\frac{1}{2}\theta)$ formulae will *not* be required. Students should be able to solve equations such as $a\cos\theta + b\sin\theta = c$ in a given interval.
5.7	**Solve simple trigonometric equations in a given interval, including quadratic equations in sin, cos and tan and equations involving multiples of the unknown angle.**	**Students should be able to solve equations such as** $\sin(x + 70°) = 0.5$ **for** $0 < x < 360°$, $3 + 5\cos 2x = 1$ **for** $-180° < x < 180°$ $6\cos^2 x + \sin x - 5 = 0, 0 \leqslant x < 360°$ These may be in degrees or radians and this will be specified in the question.
5.8	Construct proofs involving trigonometric functions and identities.	Students need to prove identities such as $\cos x \cos 2x + \sin x \sin 2x \equiv \cos x$.
5.9	Use trigonometric functions to solve problems in context, including problems involving vectors, kinematics and forces.	Problems could involve (for example) wave motion, the height of a point on a vertical circular wheel, or the hours of sunlight throughout the year. Angles may be measured in degrees or in radians.

Reciprocal Trigonometric Functions

$$\sec(x) = \frac{1}{\cos(x)}$$

Short for "**secant**"

$$\operatorname{cosec}(x) = \frac{1}{\sin(x)}$$

Short for "**cosecant**"

$$\cot(x) = \frac{1}{\tan(x)} \text{ or } \frac{\cos(x)}{\sin(x)}$$

Short for "**cotangent**"

Calculations

You have a calculator in A Level exams, but won't however in STEP, etc. It's good however to know how to calculate certain values yourself if needed.

$\cot\frac{\pi}{4} =$

$\sec\frac{\pi}{4} =$

$\operatorname{cosec}\frac{\pi}{3} =$

$\cot\frac{\pi}{6} =$

$\operatorname{cosec}\frac{5\pi}{6} =$

$\cot\frac{\pi}{3} =$

$\sec\frac{\pi}{6} =$

$\operatorname{cosec}\frac{\pi}{2} =$

$\sec\frac{5\pi}{3} =$

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