Composite Functions

Examples

1. Let $f(x) = x^2 + 1$, and $g(x) = 4x - 2$. Find

$a)\ fg(2)$ $b)\ fg(x)$

$c)\ gf(x)$ $d)\ f^2(x)$

Solve $gf(x) = 38$

2. The functions f and g are defined by

$$f: x \to |2x - 8| g: x \to \frac{x+1}{2}$$

a) Find $fg(3)$ b) Solve $fg(x) = x$

Test your understanding

1. The functions and f and g are defined by

$$f: x \to 2|x| + 3, \quad x \in \mathbb{R}$$
$$g: x \to 3 - 4x, \quad x \in \mathbb{R}$$

a) Find $fg(1)$

b) Solve the equation

$$gg(x) + [g(x)]^2 = 0$$

2. The functions f and g are defined by

$$f: x \to e^x + 2, \quad x \in \mathbb{R} g: x \to \ln x, \qquad x > 0$$

a) Find $fg(x)$, giving your answer in its simplest form.

Extension

[MAT 2014 1F]
The functions S and T are defined for real numbers by $S(x) = x + 1$ and $T(x) = -x$.
The function S is applied s times and the function T is applied t times, in some order, to produce the function

$$F(x) = 8 - x$$

It is possible to deduce that:

i) $s = 8$ and $t = 1$
ii) s is odd and t is even.
iii) s is even and t is odd.
iv) s and t are powers of 2.
v) none of the above.

[MAT 2012 Q2]
Let $f(x) = x + 1$ and $g(x) = 2x$.
i) Show that $f^2 g(x) = gf(x)$
ii) Note that $gf^2 g(x) = 4x + 4$
Find all the other ways of combining f and g that result in the function $4x + 4$.
iii) Let $i, j, k \geq 0$ be integers. Determine the function

$$f^i g f^j g f^k(x)$$

iv) Let $m \geq 0$ be an integer. How many different ways of combining the functions f and g are there that result in the function $4x + 4m$?