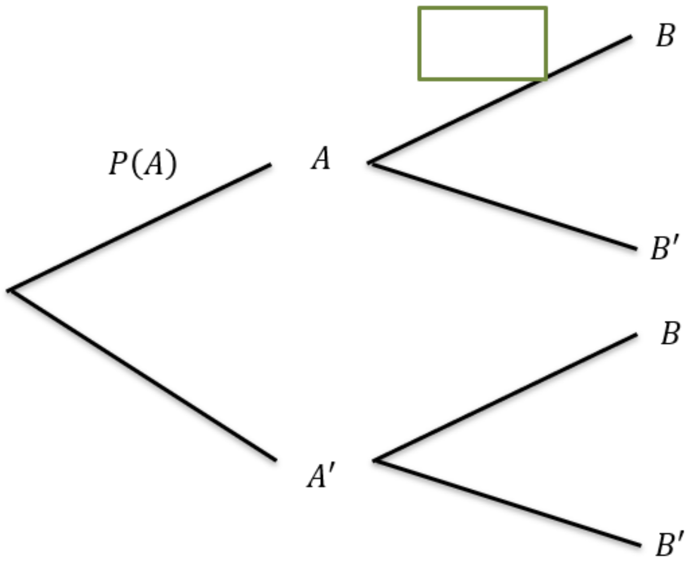
Conditional Probability

Think about how we formed a probability tree at GCSE:



$P(A \cap B) =$

Alternatively (and more commonly):

$$P(B|A) =$$

Examples

1 The following two-way table shows what foreign language students in Year 9 study.
B is the event that the student is a boy. F is the event they chose French as their language.

	B	B'	Total
F	14	38	52
F'	26	22	48
Total	40	60	100

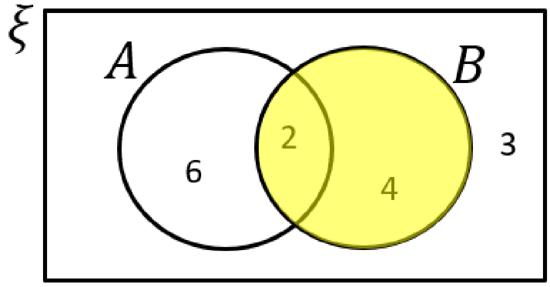
a Determine the probability of:
$P(F|B')$

Method 1: Using the formula:

Method 2: Restricted sample space.

b $P(B|F') =$

2 Using the Venn Diagram, determine:



a $P(A|B)$

Method 1: Using the formula

Method 2: Restricted sample space

b $P(A'|B') =$

c $P(B|A \cup B) =$

Further Examples

a Given that $P(A) = 0.5$ and $P(A \cap B) = 0.3$, what is $P(B|A)$?

b Given that $P(Y) = 0.6$ and $P(X \cap Y) = 0.4$, what is $P(X'|Y)$?
(Hint: Drawing a Venn Diagram will help!)

c Given that $P(A) = 0.5$, $P(B) = 0.5$ and $P(A \cap B) = 0.4$, what is $P(B|A')$?

Check your understanding

The events E and F are such that

$$P(E) = 0.28 \quad P(E \cup F) = 0.76 \qquad P(E \cap F') = 0.11$$

Find

a) $P(E \cap F) =$

b) $P(F) =$

c) $P(E'|F') =$

Further Practice

1

$$P(A \cap B') = 0.4, P(A \cup B) = 0.75$$

Then:

$$P(B) =$$

$$P(A' \cap B') =$$

2 $P(A) = 0.47$ and $P(A \cap B) = 0.12$ and $P(A' \cap B') = 0.03$

Then:

$$P(A|B') =$$

3

$$P(A') = 0.7, P(B') = 0.2, P(A \cap B') = 0.1$$

Then:

$$P(A \cup B') =$$

$$P(B|A') =$$

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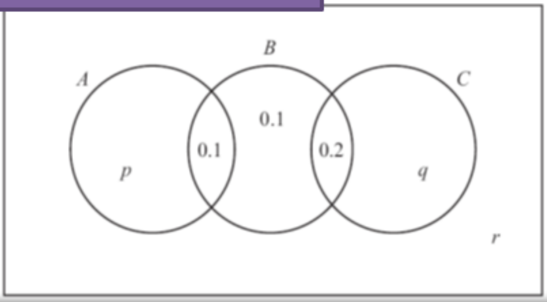

The Venn diagram in Figure 1 shows three events A, B and C and the probabilities associated with each region of B. The constants p, q and r each represent probabilities associated with the three separate regions outside B.

The events A and B are independent.

(a) Find the value of p. **(3)**

Given that $P(B|C) = \frac{5}{11}$,

(b) find the value of q and the value of r (4)

(c) Find $P(A \cup C|B)$ (2)

(a) **(From earlier)**

$$0.1 = (p + 0.1) \times 0.4$$
$$p + 0.1 = 0.25$$
$$p = 0.15$$

(b)

(c)