Upper 6 Chapter 2

Functions and Graphs

Chapter Overview

1. The Modulus Function

2. Mappings vs Functions, Domain and Range

3. Composite Functions

4. Inverse Functions

5. Transformations of the form $y = |f(x)|$ or $y = f(|x|)$. Combined transformations and transforming the modulus function.

6. Solving modulus problems

2 **Algebra and functions** *continued*	2.7	**Understand and use graphs of functions; sketch curves defined by simple equations including polynomials**	**Graph to include simple cubic and quartic functions,** **e.g. sketch the graph with equation** $y = x^2(2x-1)^2$
		The modulus of a linear function.	Students should be able to sketch the graphs of $y = \lvert ax + b \rvert$ They should be able to use their graph. For example, sketch the graph with equation $y = \lvert 2x - 1 \rvert$ and use the graph to solve the equation $\lvert 2x - 1 \rvert = x$ or the inequality $\lvert 2x - 1 \rvert > x$
		$y = \frac{a}{x}$ and $y = \frac{a}{x^2}$ **(including their vertical and horizontal asymptotes)**	**The asymptotes will be parallel to the axes e.g. the asymptotes of the curve with equation** $y = \frac{2}{x+a} + b$ **are the lines with equations** $y = b$ **and** $x = -a$
		Interpret algebraic solution of equations graphically; use intersection points of graphs to solve equations.	
		Understand and use proportional relationships and their graphs.	**Direct proportion between two variables.** **Express relationship between two variables using proportion "$\propto$" symbol or using equation involving constant** **e.g. the circumference of a semicircle is directly proportional to its diameter so** $C \propto d$ **or** $C = kd$ **and the graph of** C **against** d **is a straight line through the origin with gradient** k**.**

2 **Algebra and functions** *continued*	2.8	Understand and use composite functions; inverse functions and their graphs.	The concept of a function as a one-one or many-one mapping from $\mathbb{R}$ (or a subset of $\mathbb{R}$) to $\mathbb{R}$. The notation $\mathrm{f}: x \mapsto$ and $\mathrm{f}(x)$ will be used. Domain and range of functions. Students should know that fg will mean 'do g first, then f' and that if f^{-1} exists, then $\mathrm{f}^{-1}\mathrm{f}(x) = \mathrm{ff}^{-1}(x) = x$ They should also know that the graph of $y = \mathrm{f}^{-1}(x)$ is the image of the graph of $y = \mathrm{f}(x)$ after reflection in the line $y = x$
	2.9	**Understand the effect of simple transformations on the graph of $y = \mathrm{f}(x)$, including sketching associated graphs:** $y = a\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$, $y = \mathrm{f}(ax)$ and combinations of these transformations	Students should be able to find the graphs of $y = \lvert\mathrm{f}(x)\rvert$ and $y = \lvert\mathrm{f}(-x)\rvert$, given the graph of $y = \mathrm{f}(x)$. Students should be able to apply a combination of these transformations to any of the functions in the A Level specification (quadratics, cubics, quartics, reciprocal, $\frac{a}{x^2}$, $\lvert x\rvert$, $\sin x$, $\cos x$, $\tan x$, e^x and a^x) and sketch the resulting graph. Given the graph of $y = \mathrm{f}(x)$, students should be able to sketch the graph of, e.g. $y = 2\mathrm{f}(3x)$, or $y = \mathrm{f}(-x) + 1$, and should be able to sketch (for example) $y = 3 + \sin 2x$, $y = -\cos\left(x + \frac{\pi}{4}\right)$
2 **Algebra and functions** *continued*	2.11	Use of functions in modelling, including consideration of limitations and refinements of the models.	For example, use of trigonometric functions for modelling tides, hours of sunlight, etc. Use of exponential functions for growth and decay (see Paper 1, Section 6.7). Use of reciprocal function for inverse proportion (e.g. pressure and volume).

The Modulus Function

Example:

1. If $f(x) = |2x - 3| + 1$, find

a) $f(5)$

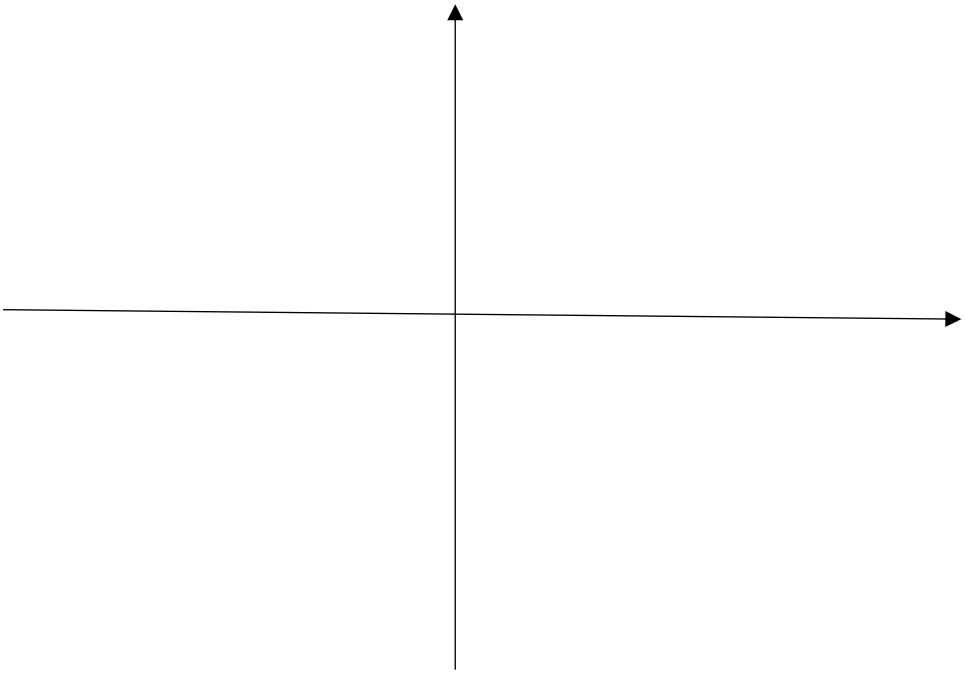
b) $f(-2)$

c) $f(1)$

Modulus Graphs

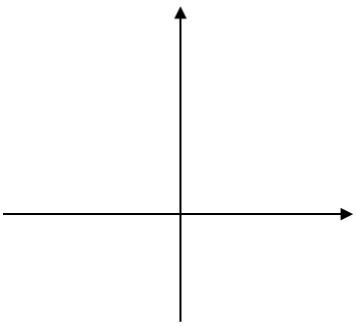
$y = |x|$

x	-2	-1	0	1	2
y					

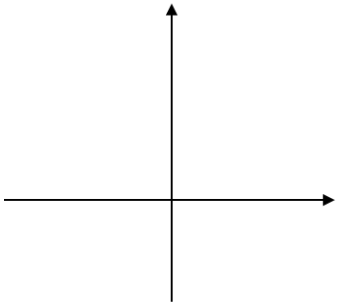


Examples

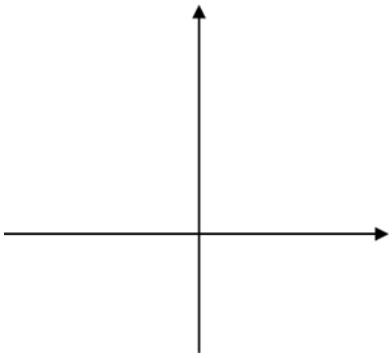
1. Sketch $y = |2x - 3|$



2. Solve $|2x - 3| = 5$

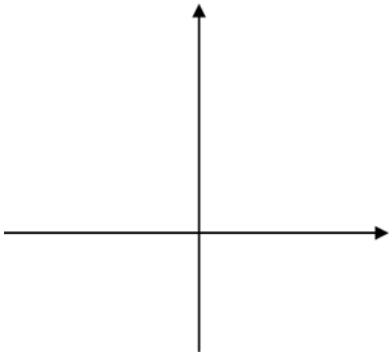


3. Solve $|3x - 5| = 2 - \frac{1}{2}x$



Test Your Understanding

1. Solve $|x + 1| = 2x + 5$



2. Solve $|4x - 1| < 2x$