Stats Yr2 Chapter 2 :: Conditional Probability

Chapter Overview

1:: Set Notation

How sets are used to describe a sample space/event and how notation like $A \cap B$ is used to combine sets.

2:: Conditional Probability in Venn Diagrams

The notation $P(A|B)$ means "the probability of A given that B happened". How we can find such probabilities using a Venn Diagram.

3:: Formula for Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

4:: Tree Diagrams

"I have 3 red and 4 green balls in a bag. I take one ball out the bag, keep it, then take another. **Given that** the second ball was green, determine the probability the first was red."
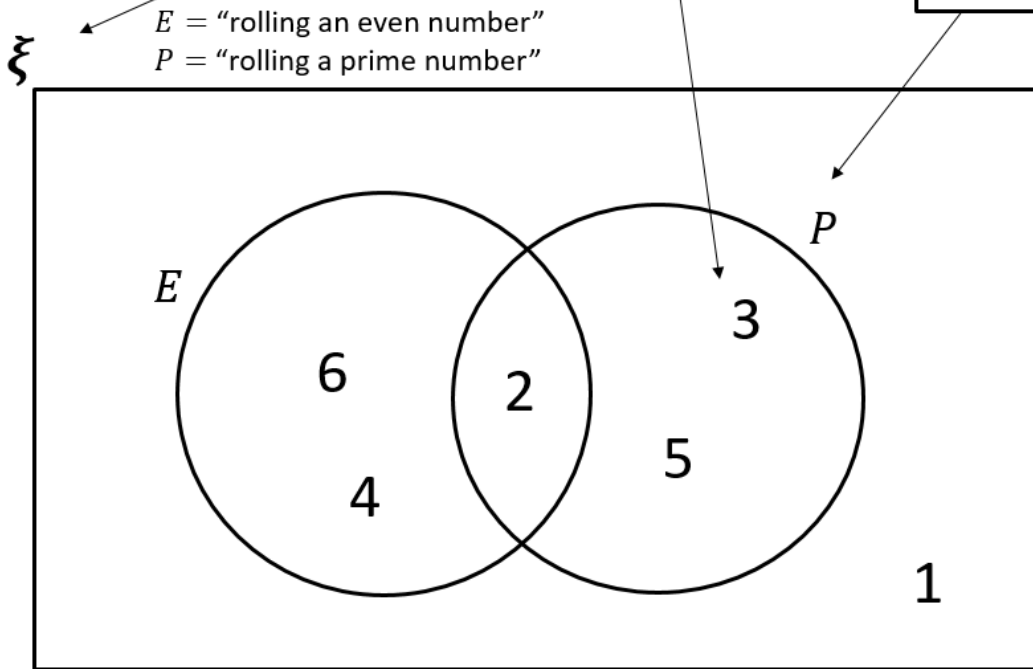
RECAP :: Using sets for sample spaces and events

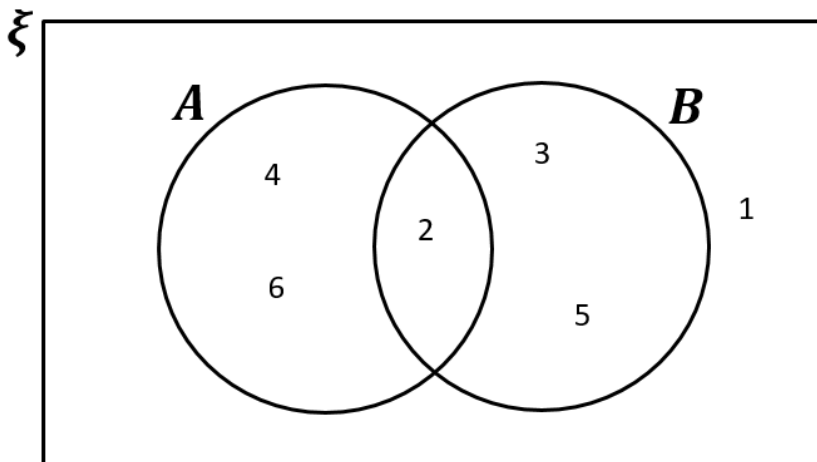In general, sets are used to represent **collections of items**.

A **sample space** is **set of all possible outcomes**. We use ξ (Greek 'Xi'), or sometimes just S, to represent this set. We use a rectangle in a Venn Diagram.

Each number represents an **outcome**.

In probability, an **event** is a **set of one or more outcomes**. These are the circles in the Venn Diagram. We use capital letters for the variables representing sets.

E = "rolling an even number"
P = "rolling a prime number"

ξ

E 6   2   3 P
       4   5
                      1
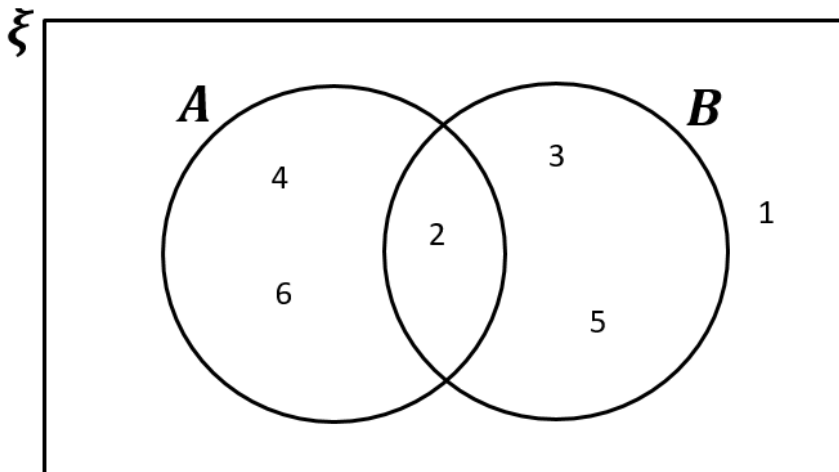
Combining events/sets

ξ = the whole sample space (1 to 6)

A = even number on a die thrown

B = prime number on a die thrown

	What does it mean in this context?	What is the resulting set of outcomes?
A'		
$A \cup B$		
$A \cap B$		

Some fundamentals

S = the whole sample space (1 to 6)

A = even number on a die thrown

B = prime number on a die thrown

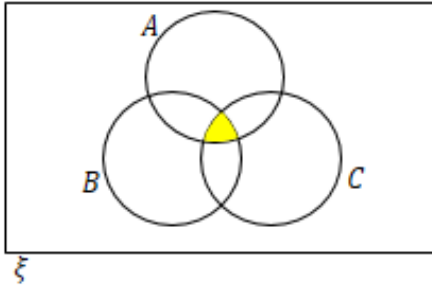|	What does it mean in this context?	What is the resulting set of outcomes?
$A \cap B'$		
$(A \cup B)'$		
$(A \cap B)'$		

What area is indicated?
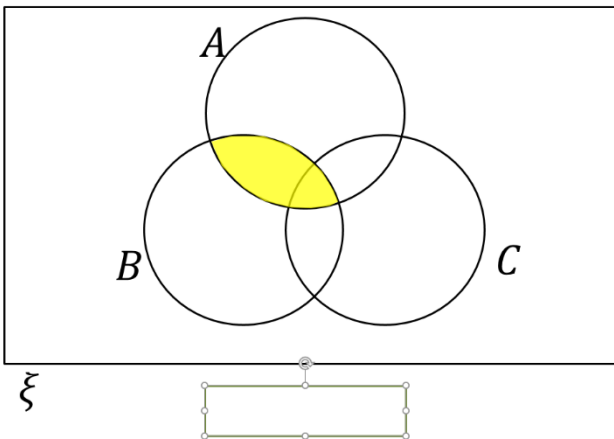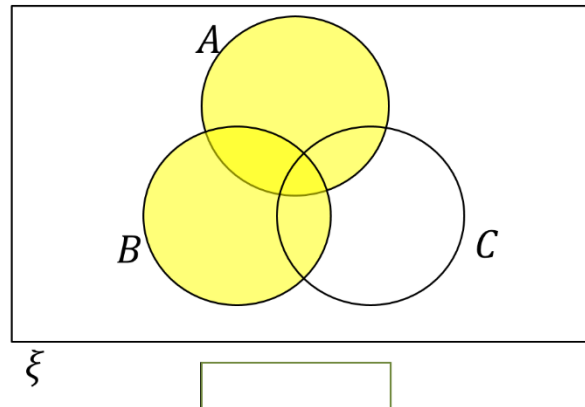
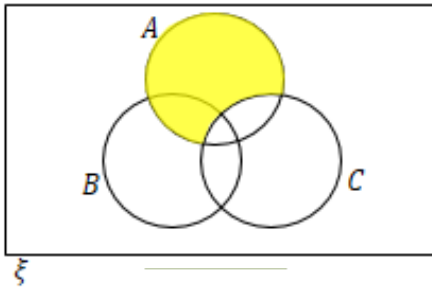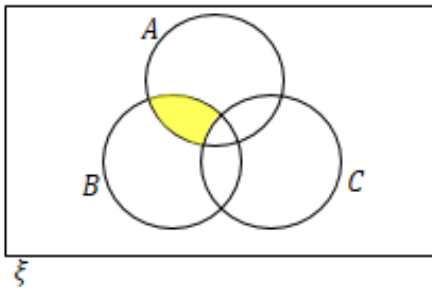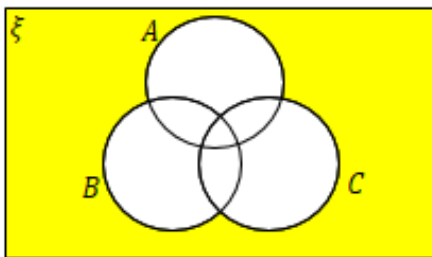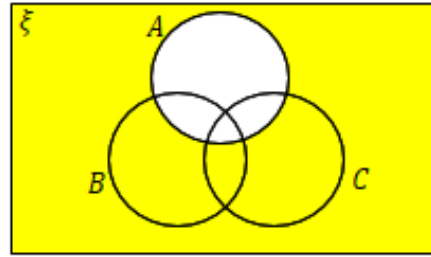What area is indicated?

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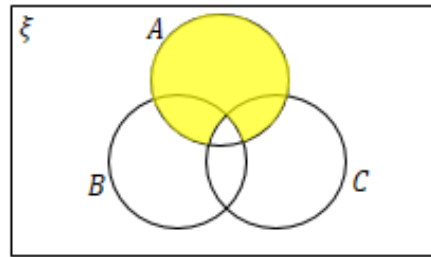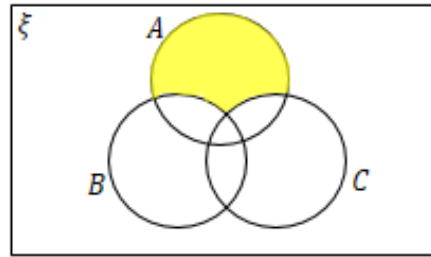What area is indicated?

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What area is indicated?

Examples

Venn Diagram can either contain:
(a) The **specific outcomes** in each set
(b) The number of items in the set (i.e. **frequencies**)
(c) The **probability** of being in that set.

[Textbook] A card is selected at random from a pack of 52 playing cards. Let A be the event that the card is an ace and D the event that the card is a diamond. Find:
a) $P(A \cap D)$ b) $P(A \cup D)$ c) $P(A')$ d) $P(A' \cap D)$
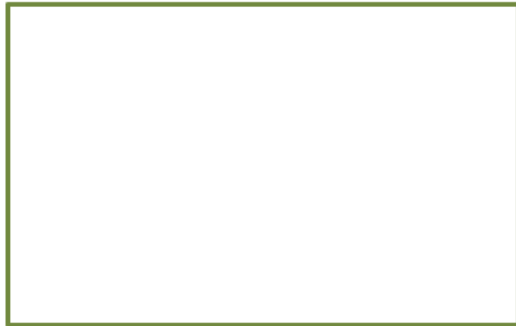
Examples

[Textbook] Given that $P(A) = 0.3$, $P(B) = 0.4$ and $P(A \cap B) = 0.25$,
a. Explain why events A and B are not independent.
Given also that $P(C) = 0.2$, that events A and C are mutually exclusive and that events B and C are independent,
b. Draw a Venn diagram to illustrate the events A, B and C, showing the probabilities for each region.
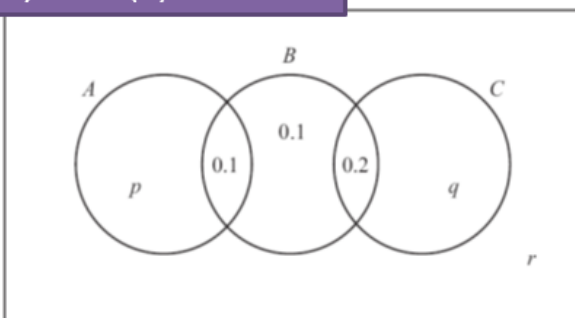c. Find $P\big((A \cap B') \cup C\big)$

a

b

c

Test Your Understanding

(a)

The Venn diagram in Figure 1 shows three events A, B and C and the probabilities associated with each region of B. The constants p, q and r each represent probabilities associated with the three separate regions outside B.

The events A and B are independent.

(a) Find the value of p.   (3)