U6 Chapter 1

Algebraic Methods

Chapter Overview

1. Proof by contradiction

2. Algebraic fractions

3. Partial fractions

4. Algebraic division

Course specification

	Disproof by counter example Proof by contradiction (including proof of the irrationality of √2 and the infinity of primes, and application to unfamiliar proofs).	**Disproof by counter example** **e.g. show that the statement "$n^2 - n + 1$ is a prime number for all values of n" is untrue**
2.6	**Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the factor theorem.** Simplify rational expressions, including by factorising and cancelling, and algebraic division (by linear expressions only).	**Only division by $(ax + b)$ or $(ax - b)$ will be required. Students should know that if $f(x) = 0$ when $x = a$, then $(x - a)$ is a factor of $f(x)$.** **Students may be required to factorise cubic expressions such as $x^3 + 3x^2 - 4$ and $6x^3 + 11x^2 - x - 6$.** Denominators of rational expressions will be linear or quadratic, e.g. $\frac{1}{ax+b}, \frac{ax+b}{px^2+qx+r}, \frac{x^3+a^3}{x^2-a^2}$
2.10	Decompose rational functions into partial fractions (denominators not more complicated than squared linear terms and with no more than 3 terms, numerators constant or linear).	Partial fractions to include denominators such as $(ax + b)(cx + d)(ex + f)$ and $(ax + b)(cx + d)^2$. Applications to integration, differentiation and series expansions.

1 :: Proof By Contradiction

✐ To prove a statement is true by contradiction:
- **Assume** that the statement is in fact **false.**
- Prove that this would **lead to a contradiction**.
- Therefore we were wrong in assuming the statement was false, and therefore it must be true.

Prove that there is no greatest odd integer.

Assume that there is a greatest odd integer, n.

Then $n + 2$ is an odd integer which is larger than n.
This contradicts the assumption that n is the greatest odd integer.

Therefore, there is no greatest odd integer.

How to structure/word proof:
1. *"Assume that [negation of statement]."*
2. *[Reasoning followed by…] "This contradicts the assumption that…"* or *"This is a contradiction"*.
3. "Therefore [*restate original statement*]."

Negating the original statement

The first part of a proof by contradiction requires you to negate the original statement. What is the negation of each of these statements? ***(Click to choose)***

"There are infinitely many prime numbers."

"All Popes are Catholic."

"If it is raining, my garden is wet."

More Examples

Prove by contradiction that if n^2 is even, then n must be even.

Prove by contradiction that $\sqrt{2}$ is an irrational number.

Prove by contradiction that there are infinitely many prime numbers.