## U6 Chapter 1

# **Algebraic Methods**

#### **Chapter Overview**

- 1. Proof by contradiction
- 2. Algebraic fractions
- 3. Partial fractions
- 4. Algebraic division

## **Course specification**

	Disproof by counter example Proof by contradiction (including proof of the irrationality of √2 and the infinity of primes, and application to unfamiliar	Disproof by counter example e.g. show that the statement " $n^2 - n + 1$ is a prime number for all values of $n''$ is untrue
2.6	proofs). Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the factor theorem. Simplify rational expressions,	Only division by $(ax + b)$ or $(ax - b)$ will be required. Students should know that if $f(x) = 0$ when $x = a$ , then $(x - a)$ is a factor of $f(x)$ . Students may be required to factorise cubic expressions such as $x^3 + 3x^2 - 4$ and $6x^3 + 11x^2 - x - 6$ . Denominators of rational expressions will
	including by factorising and cancelling, and algebraic division (by linear expressions only).	be linear or quadratic, e.g. $\frac{1}{ax+b}$ , $\frac{ax+b}{px^2+qx+r}$ , $\frac{x^3+a^3}{x^2-a^2}$
2.10	Decompose rational functions into partial fractions (denominators not more complicated than squared linear terms and with no more than 3 terms, numerators constant or linear).	Partial fractions to include denominators such as (ax + b)(cx + d)(ex + f) and $(ax + b)(cx + d)^2$ . Applications to integration, differentiation and series expansions.

#### 1 :: Proof By Contradiction

✓ To prove a statement is true by contradiction:

- Assume that the statement is in fact false.
- Prove that this would lead to a contradiction.
- Therefore we were wrong in assuming the statement was false, and therefore it must be true.

Prove that there is no greatest odd integer.

Assume that there is a greatest odd integer, n.

Then n + 2 is an odd integer which is larger than n.

This contradicts the assumption that n is the greatest odd integer.

Therefore, there is no greatest odd integer.

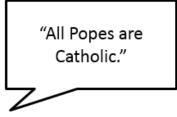
#### How to structure/word proof:

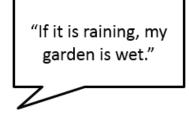
- "Assume that [negation of statement]."
- 2. [Reasoning followed by...] "This contradicts the assumption that..." or "This is a contradiction".
- 3. "Therefore [restate original statement]."

## Negating the original statement

The first part of a proof by contradiction requires you to negate the original statement. What is the negation of each of these statements? *(Click to choose)* 

"There are infinitely		
many prime		
numbers."		
$\overline{}$		





# More Examples

Prove by contradiction that if  $n^2$  is even, then n must be even.

Prove by contradiction that  $\sqrt{2}$  is an irrational number.

Prove by contradiction that there are infinitely many prime numbers.