THE NEWTON-RAPHSON METHOD

The Newton- Raphson method can be used to find numerical solutions to equations of the form $\boldsymbol{f(x) = 0}$. You need to be able to differentiate $\boldsymbol{f(x)}$ in order to use this method.

The Newton- Raphson formula is:

$$\boldsymbol{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

Example 1

Recall that in lesson 1 we saw that the function $\boldsymbol{f(x) = x - \cos(x)}$ has a root, α, in the interval $0 < \alpha < 1$.

Using $x_0 = 0.5$ as a first approximation to α, apply the Newton-Raphson procedure three times to find a better approximation to α which, in this case, will be accurate to 7 decimal places.

To perform iterations quickly, do the following on your calculator:

[0.5] [=]

[ANS] – (ANS – cos(ANS))/(1 + sin(ANS))

Then press [=].

Example 2 Edexcel FP1 June 2013(R) Q3c

$$f(x) = \frac{1}{2}x^4 - x^3 + x - 3$$

...

The equation $f(x) = 0$ has a root β in the interval $[-2, -1]$.

(c) Taking -1.5 as a first approximation to β, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to β.
Give your answer to 2 decimal places.

(5)

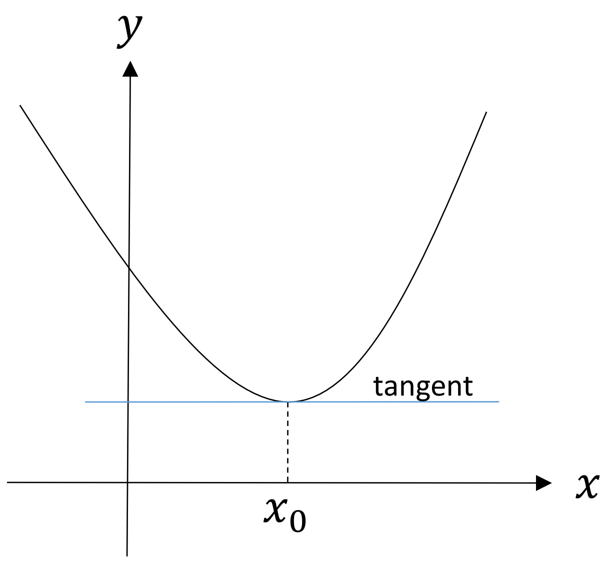
Example 3 Edexcel FP1 Jan 2010 Q2c

$$f(x) = 3x^2 - \frac{11}{x^2}.$$

(c) Taking 1.4 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to obtain a second approximation to α, giving your answer to 3 decimal places.

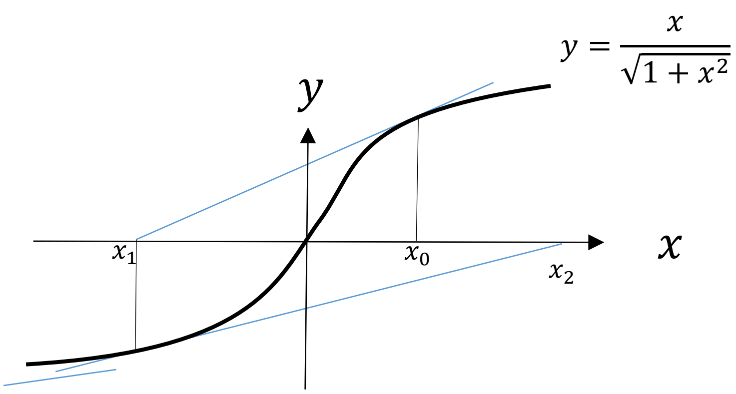
(5)

When does Newton-Raphson fail?



$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

If the starting value x_0 was the stationary point, then $f'(x_0) = 0$, resulting in a division by 0 in the above formula.
Graphically, it is because the tangent will never reach the x-axis.



Newton-Raphson also suffers from the same drawbacks as solving by iteration, in that it is possible for the values of x_i to **diverge**.
In this example, the x_i oscillate either side of 0, but get gradually further away from $\alpha = 0$.

Exercise 10C Page 284