**THE NEWTON-RAPHSON METHOD**

**The Newton- Raphson method can be used to find numerical solutions to equations of the form** $f\left(x\right)=0$**. You need to be able to differentiate** $f\left(x\right)$ **in order to use this method.**

**The Newton- Raphson formula is:**

$$x\_{n+1}=x\_{n}-\frac{f\left(x\_{n}\right)}{f^{'}\left(x\_{n}\right)}$$

**Example 1**

Recall that in lesson 1 we saw that the function $f\left(x\right)=x-\cos(\left(x\right))$has aroot, $α$, in the interval $0<α<1$.

Using $x\_{0}=0.5$ as a first approximation to$ α$, apply the Newton-Raphson procedure three times to find a better approximation to $α$ which, in this case, will be accurate to 7 decimal places.

To perform iterations quickly, do the following on your calculator:

[0.5] [=]

[ANS] – (ANS – cos(ANS))/(1 + sin(ANS))

Then press [=].

**Example 2**

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**Example 3**

**When does Newton-Raphson fail?**

$$x$$

$$y$$

$$x\_{0}$$

tangent

$$x\_{n+1}=x\_{n}-\frac{f\left(x\_{n}\right)}{f^{'}\left(x\_{n}\right)}$$

**If the starting value** $x\_{0}$ **was the stationary point**, then $f^{'}\left(x\_{0}\right)=0$, resulting in a division by 0 in the above formula.

Graphically, it is because the tangent will never reach the $x$-axis.

$$x$$

$$y$$

$$y=\frac{x}{\sqrt{1+x^{2}}}$$

$$x\_{0}$$

$$x\_{1}$$

$$x\_{2}$$

Newton-Raphson also suffers from the same drawbacks as solving by iteration, in that it is possible for the values of $x\_{i}$ to **diverge**.

In this example, the $x\_{i}$ oscillate either side of 0, but get gradually further away from $α=0$.

Exercise 10C Page 284