ITERATION

To solve $f(x) = 0$ by an iterative method, rearrange into a form $x = g(x)$ and use the iterative formula $x_{n+1} = g(x_n)$

Example 1 Edexcel C3 Jan 2013

$$g(x) = e^{x-1} + x - 6$$

(*a*) Show that the equation $g(x) = 0$ can be written as

$$x = \ln(6 - x) + 1, \qquad x < 6.$$

(2)

The root of $g(x) = 0$ is α.

The iterative formula

$$x_{n+1} = \ln(6 - x_n) + 1, \qquad x_0 = 2.$$

is used to find an approximate value for α.

(*b*) Calculate the values of x_1, x_2 and x_3 to 4 decimal places.

(3)

(*c*) By choosing a suitable interval, show that $\alpha = 2.307$ correct to 3 decimal places.

(3)

a)

b) **x_1, x_2, x_3 represent successively better approximations of the root**

Initially type x_0 (i.e. 2) onto your calculator.
Now just type: $\ln(6 - ANS) + 1$
And then press your $=$ key to get successive iterations.

c)

The starting value $x_0$matters.

- If there are a multiple roots, the iteration might converge to (i.e. approach) a different root.
- The iteration not converge to a root at all and **diverges** (i.e. approach infinity).

Example 2

$$f(x) = x^3 - 3x^2 - 2x + 5$$

(a) Show that the equation $f(x) = 0$ has a root in the interval $3 < x < 4$.

(b) Use the iterative formula $x_{n+1} = \sqrt{\frac{x_n^3 - 2x_n + 5}{3}}$ to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places, and taking:
(i) $x_0 = 1.5$ (ii) $x_0 = 4$

Staircase and cobweb diagrams

Example 3

$$f(x) = x^2 - 8x + 4$$

(a) Show that the root of the equation $f(x) = 0$ can be written as $x = \sqrt{8x - 4}$

(b) Using the iterative formula $x_{n+1} = \sqrt{8x_n - 4}$, and starting with $x_0 = 1$, draw a staircase diagram, indicating x_0, x_1, x_2 on your x-axis, as well as the root α.

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