1 **a** $4\sin x = -\cos x$

$\frac{\sin x}{\cos x} = -\frac{1}{4}$

$\tan x = -\frac{1}{4}$

b $x = 180 - 14.0,\ 360 - 14.0$

$x = 166.0°,\ 346.0°$

2 **a** LHS $= 5\sin^2 x + 5\sin x + 4(1 - \sin^2 x)$

$= \sin^2 x + 5\sin x + 4$

$=$ RHS

b $(\sin x + 4)(\sin x + 1) = 0$

$\sin x = -1$ or -4 [no solutions]

$x = 270°$

3 **a** $2\sin x = \cos x$

$\tan x = 0.5$

$x = 26.6,\ 180 + 26.6$

$x = 26.6°,\ 206.6°$

b $\tan x = \frac{4}{3}$

$x = 53.1,\ 180 + 53.1$

$x = 53.1°,\ 233.1°$

c $1 - \sin^2 x + 3\sin x - 3 = 0$

$\sin^2 x - 3\sin x + 2 = 0$

$(\sin x - 1)(\sin x - 2) = 0$

$\sin x = 1$ or 2 [no solutions]

$x = 90°$

d $3\cos^2 x - (1 - \cos^2 x) = 2$

$4\cos^2 x = 3$

$\cos x = \pm\frac{\sqrt{3}}{2}$

$x = 30,\ 360 - 30$ or $180 - 30,\ 180 + 30$

$x = 30°,\ 150°,\ 210°,\ 330°$

e $2(1 - \cos^2 x) + 3\cos x = 3$

$2\cos^2 x - 3\cos x + 1 = 0$

$(2\cos x - 1)(\cos x - 1) = 0$

$\cos x = 0.5$ or 1

$x = 60,\ 360 - 60$ or $0,\ 360$

$x = 0,\ 60°,\ 300°,\ 360°$

f $3(1 - \sin^2 x) = 5(1 - \sin x)$

$3\sin^2 x - 5\sin x + 2 = 0$

$(3\sin x - 2)(\sin x - 1) = 0$

$\sin x = \frac{2}{3}$ or 1

$x = 41.8,\ 180 - 41.8$ or 90

$x = 41.8°,\ 90°,\ 138.2°$

g $3\sin^2 x = 8\cos x$

$3(1 - \cos^2 x) = 8\cos x$

$3\cos^2 x + 8\cos x - 3 = 0$

$(3\cos x - 1)(\cos x + 3) = 0$

$\cos x = \frac{1}{3}$ or -3 [no solutions]

$x = 70.5,\ 360 - 70.5$

$x = 70.5°,\ 289.5°$

h $\cos^2 x = 3\sin x$

$1 - \sin^2 x = 3\sin x$

$\sin^2 x + 3\sin x - 1 = 0$

$\sin x = \frac{-3 \pm \sqrt{9+4}}{2}$

$\sin x = \frac{1}{2}(-3 + \sqrt{13})$ or $\frac{1}{2}(-3 - \sqrt{13})$ [no sols]

$x = 17.6,\ 180 - 17.6$

$x = 17.6°,\ 162.4°$

i $3(1 - \cos^2 x) - 5\cos x + 2\cos^2 x = 0$

$\cos^2 x + 5\cos x - 3 = 0$

$\cos x = \frac{-5 \pm \sqrt{25+12}}{2}$

$\cos x = \frac{1}{2}(-5 + \sqrt{37})$ or $\frac{1}{2}(-5 - \sqrt{37})$ [no sols]

$x = 57.2,\ 360 - 57.2$

$x = 57.2°,\ 302.8°$

j $2\sin^2 x + 7\sin x - 2(1 - \sin^2 x) = 0$

$4\sin^2 x + 7\sin x - 2 = 0$

$(4\sin x - 1)(\sin x + 2) = 0$

$\sin x = 0.25$ or -2 [no solutions]

$x = 14.5,\ 180 - 14.5$

$x = 14.5°,\ 165.5°$

k $3\sin x = 2\tan x$

$3\sin x\cos x = 2\sin x$

$\sin x\,(3\cos x - 2) = 0$

$\sin x = 0$ or $\cos x = \frac{2}{3}$

$x = 0,\ 180,\ 360$ or $48.2,\ 360 - 48.2$

$x = 0,\ 48.2°,\ 180°,\ 311.8°,\ 360°$

l $(1 - \cos^2 x) - 9\cos x - \cos^2 x = 5$

$2\cos^2 x + 9\cos x + 4 = 0$

$(2\cos x + 1)(\cos x + 4) = 0$

$\cos x = -0.5$ or -4 [no solutions]

$x = 180 - 60,\ 180 + 60$

$x = 120°,\ 240°$

4 **a** $\cos\theta = \pm 0.5$

$\theta = \frac{\pi}{3}, -\frac{\pi}{3}$ or $\pi - \frac{\pi}{3}, -\pi + \frac{\pi}{3}$

$\theta = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$

b $(2\sin\theta + 1)^2 = 0$

$\sin\theta = -0.5$

$\theta = -\frac{\pi}{6}, -\pi + \frac{\pi}{6}$

$\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}$

c $(\cos\theta + 3)(\cos\theta - 1) = 0$

$\cos\theta = 1$ or -3 [no solutions]

$\theta = 0$

d $3\sin^2\theta - (1 - \sin^2\theta) = 0$

$4\sin^2\theta = 1$

$\sin\theta = \pm 0.5$

$\theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}$ or $-\frac{\pi}{6}, -\pi + \frac{\pi}{6}$

$\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$

e $4\sin^2\theta - 5\sin\theta + 2(1 - \sin^2\theta) = 0$

$2\sin^2\theta - 5\sin\theta + 2 = 0$

$(2\sin\theta - 1)(\sin\theta - 2) = 0$

$\sin\theta = 0.5$ or 2 [no solutions]

$\theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

f $(1 - \cos^2\theta) - 3\cos\theta - \cos^2\theta = 2$

$2\cos^2\theta + 3\cos\theta + 1 = 0$

$(2\cos\theta + 1)(\cos\theta + 1) = 0$

$\cos\theta = -0.5$ or -1

$\theta = \pi - \frac{\pi}{3}, -\pi + \frac{\pi}{3}$ or $-\pi, \pi$

$\theta = -\pi, -\frac{2\pi}{3}, \frac{2\pi}{3}, \pi$

5 **a** LHS $= \sin^2 x + 2\sin x\cos x + \cos^2 x$

$= (\sin^2 x + \cos^2 x) + 2\sin x\cos x$

$= 1 + 2\sin x\cos x$

$=$ RHS

b LHS $= \frac{1-\cos^2 x}{\cos x}$

$= \frac{\sin^2 x}{\cos x}$

$= \sin x \times \frac{\sin x}{\cos x}$

$= \sin x\tan x$

$=$ RHS

c LHS $= \frac{1-\sin^2 x}{1-\sin x}$

$= \frac{(1+\sin x)(1-\sin x)}{1-\sin x}$

$= 1 + \sin x$

$=$ RHS

d LHS $= \frac{(1+\sin x)(1-\sin x)}{\cos x(1-\sin x)}$

$= \frac{1-\sin^2 x}{\cos x(1-\sin x)}$

$= \frac{\cos^2 x}{\cos x(1-\sin x)}$

$= \frac{\cos x}{1-\sin x}$

$=$ RHS

6 **a** LHS $= \cos^2 x - 2\cos x\tan x + \tan^2 x + \sin^2 x + 2\sin x + 1$

$= \cos^2 x - 2\sin x + \tan^2 x + \sin^2 x + 2\sin x + 1$

$= (\cos^2 x + \sin^2 x) + \tan^2 x + 1$

$= 2 + \tan^2 x =$ RHS

b $2 + \tan^2 x = 3$

$\tan^2 x = 1$

$\tan x = \pm 1$

$x = \frac{\pi}{4}, \pi + \frac{\pi}{4}$ or $\pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

7 **a** $f(x) = (1 - \sin^2 x) + 2\sin x$

$= 2 - (\sin^2 x - 2\sin x + 1)$

$= 2 - (\sin x - 1)^2$

b max. value of $f(x) = 2$

occurs when $\sin x = 1 \;\therefore\; x = \frac{\pi}{2}$