U6 Pure Chapter 5

Radians

Course Structure

1: Converting between degrees and radians.

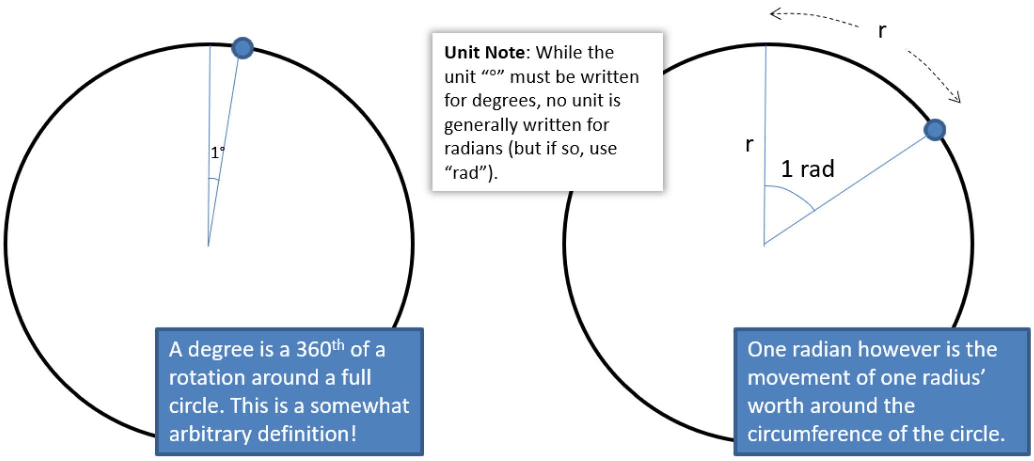
2: Find arc length and sector area (when using radians)

3: Solve trig equations in radians.

4: Small angle approximations

5 **Trigonometry**	5.1	**Understand and use the definitions of sine, cosine and tangent for all arguments;** **the sine and cosine rules;** **the area of a triangle in the form $\frac{1}{2}ab\sin C$** Work with radian measure, including use for arc length and area of sector.	**Use of x and y coordinates of points on the unit circle to give cosine and sine respectively,** **including the ambiguous case of the sine rule.** Use of the formulae $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ for arc lengths and areas of sectors of a circle.
	5.2	Understand and use the standard small angle approximations of sine, cosine and tangent $\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{\theta^2}{2}$, $\tan\theta \approx \theta$ Where θ is in radians.	Students should be able to approximate, e.g. $\frac{\cos 3x - 1}{x\sin 4x}$ when x is small, to $-\frac{9}{8}$

Radians



Converting between radians and degrees

$90° =$

$135° =$

$\frac{\pi}{3} =$

$\frac{3}{2}\pi =$

$45° =$

$72° =$

$\frac{\pi}{6} =$

$\frac{5\pi}{6} =$

It is useful to <u>remember</u> the standard angle conversions….

$45° =$

$30° =$

$60° =$

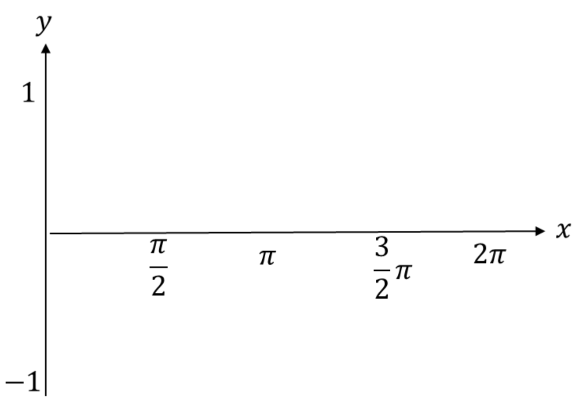
$135° =$

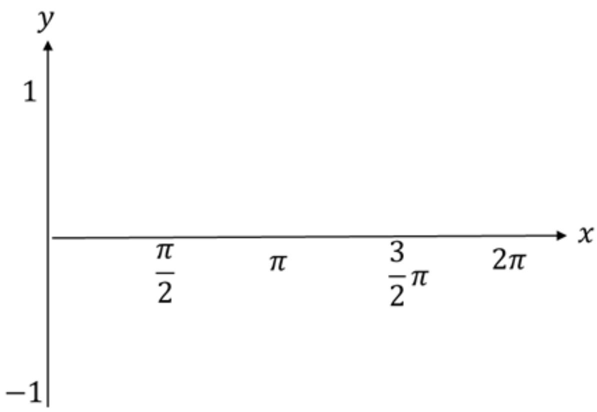
$270° =$

$90° =$

$120° =$

<u>Graph Sketching with Radians</u>





Test Your Understanding

Sketch the graph of $y = \cos\left(x + \frac{\pi}{2}\right)$ for $0 \leq x < 2\pi$

Sin, cos, tan of angles in radians

Reminder of laws from Year 1:

- $\sin(x) = \sin(180 - x)$
- $\cos(x) = \cos(360 - x)$
- sin, cos repeat every $360°$ but tan every $180°$

In terms of radians:

- $\sin(x) =$
- $\cos(x) =$
- sin, cos repeat every ________ but tan every _____.

To find sin/cos/tan of a ‘**common**’ angle in radians without using a calculator, it is easiest to just **convert to degrees first**.

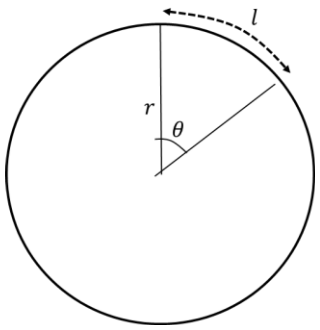
Examples

1. $\cos\left(\frac{4\pi}{3}\right) =$

2. $\sin\left(-\frac{7\pi}{6}\right) =$

“Use of Technology” :
To find $\cos\left(\frac{4\pi}{3}\right)$ directly using your calculator, you need to switch to radians mode. Press $SHIFT \rightarrow SETUP$, then $ANGLE\ UNIT$, then $Radians$. An R will appear at the top of your screen, instead of D.

Arc length



Arc length in degrees =

Arc length in radians =

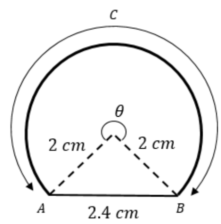
Examples

1. Find the length of the arc of a circle of radius 5.2 cm, given that the arc subtends an angle of 0.8 radians at the centre of the circle.

2. An arc AB of a circle with radius 7 cm and centre O has a length of 2.45 cm. Find the angle $\angle AOB$ subtended by the arc at the centre of the circle

3. An arc AB of a circle, with centre O and radius r cm, subtends an angle of θ radians at O. The perimeter of the sector AOB is P cm. Express r in terms of P and θ.

4. The border of a garden pond consists of a straight edge AB of length 2.4m, and a curved part C, as shown in the diagram. The curve part is an arc of a circle, centre O and radius 2m.
Find the length of C.



Test Your Understanding

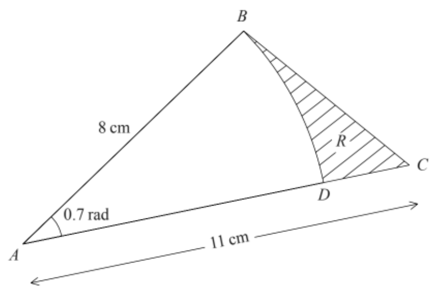
Figure 1 shows the triangle ABC, with $AB = 8\ cm$, $AC = 11\ cm$ and $\angle BAC = 0.7$ radians. The arc BD, where D lies on AC, is an arc of a circle with centre A and radius 8 cm. The region R, shown shaded in Figure 1, is bounded by the straight lines BC and CD and the arc BD.

Find

(a) The length of the arc BD.

(b) The perimeter of R, giving your answer to 3 significant figures.

Figure 1



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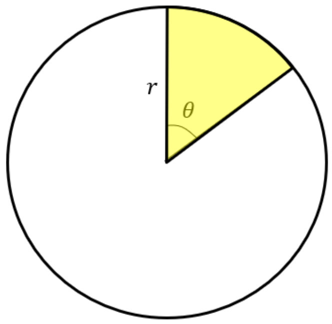
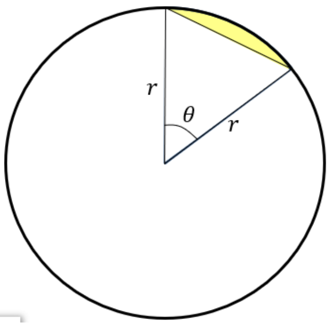
Sector Area



Area using Degrees =

Area using Radians =

Segment Area



A segment is the region bound between a chord and the circumference.

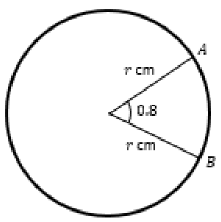
This is just a sector with a triangle cut out.

Recall that the area of a triangle is $\frac{1}{2}ab\sin C$ where C is the 'included angle' (i.e. between a and b)

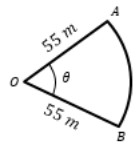
Area using radians:

Examples

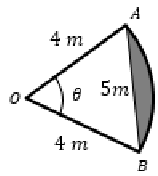
1. In the diagram, the area of the minor sector AOB is 28.9 cm^2. Given that $\angle AOB = 0.8$ radians, calculate the value of r.



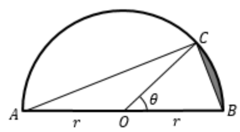
2. A plot of land is in the shape of a sector of a circle of radius 55 m. The length of fencing that is erected along the edge of the plot to enclose the land is 176 m. Calculate the area of the plot of land.



3. In the diagram above, OAB is a sector of a circle, radius 4m. The chord AB is 5m long. Find the area of the shaded segment.



4. In the diagram, AB is the diameter of a circle of radius $r$cm, and $\angle BOC = \theta$ radians. Given that the area of ΔAOC is three times that of the shaded segment, show that
$3\theta - 4\sin\theta = 0.$



Test Your Understanding

6.

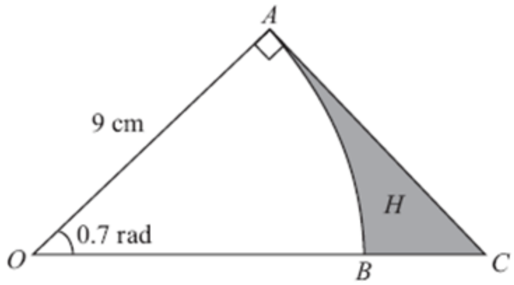


Figure 1

Figure 1 shows the sector OAB of a circle with centre O, radius 9 cm and angle 0.7 radians.

(a) Find the length of the arc AB.

(2)

(b) Find the area of the sector OAB.

(2)

The line AC shown in Figure 1 is perpendicular to OA, and OBC is a straight line.

(c) Find the length of AC, giving your answer to 2 decimal places.

(2)

The region H is bounded by the arc AB and the lines AC and CB.

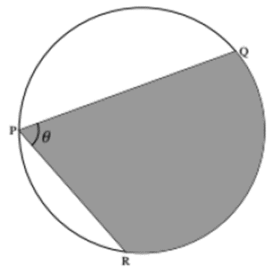
(d) Find the area of H, giving your answer to 2 decimal places.

(3)

Extension

[MAT 2012 1J]

If two chords QP and RP on a circle of radius 1 meet in an angle θ at P, for example as drawn in the diagram on the left, then find the largest possible area of the shaded region RPQ, giving your answer in terms of θ.



Solving Trigonometric Equations

Solving trigonometric equations is virtually the same as you did in Year 1, except:

(a) Your calculator needs to be in radians mode.
(b) We use $\pi -$ instead of $180° -$, and so on.

Remember

- $\sin(x) = \sin(\pi - x)$
- $\cos(x) = \cos(2\pi - x)$
- sin, cos repeat every 2π but tan every π

Example
Solve the equation

$\sin 3\theta = \frac{\sqrt{3}}{2}$ in the interval $0 \leq \theta \leq 2\pi$.

Test Your Understanding

[Jan 07 Q6]
Find all the solutions, in the interval $0 \leq x < 2\pi$, of the equation
$2\cos^2 x + 1 = 5\sin x$, giving each solution in terms of π. **(6)**

Extension

[MAT 2010 1C] In the range $0 \leq x \leq 2\pi$, the equation $\sin^2 x + 3\sin x\cos x + 2\cos^2 x = 0$ has how many solutions?

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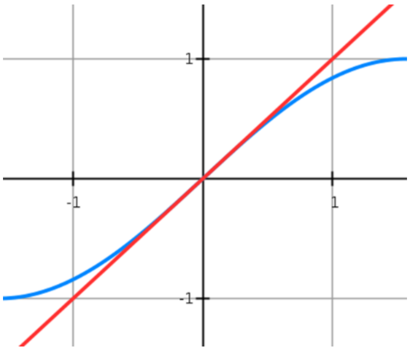
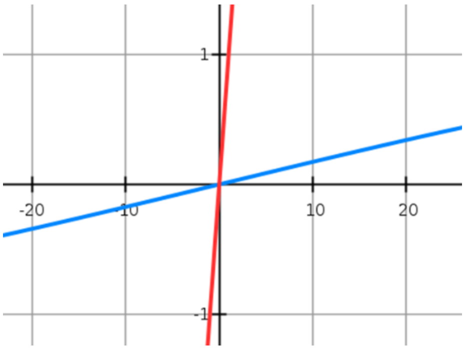
Small Angle Approximations





Example

When θ is small, find the approximate value of:

a) $\frac{\sin 2\theta + \tan \theta}{2\theta}$

b) $\frac{\cos 4\theta - 1}{\theta \sin 2\theta}$

Example

a) Show that, when θ is small,

$$\sin 5\theta + \tan 2\theta - \cos 2\theta \approx 2\theta^2 + 7\theta - 1$$

b) Hence state the approximate value of
$\sin 5\theta + \tan 2\theta - \cos 2\theta$ for small values of θ.

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