U6 Pure Chapter 5

Radians

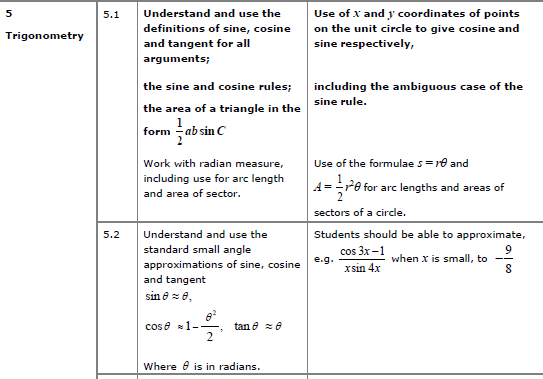
Course Structure

**1**: Converting between degrees and radians.

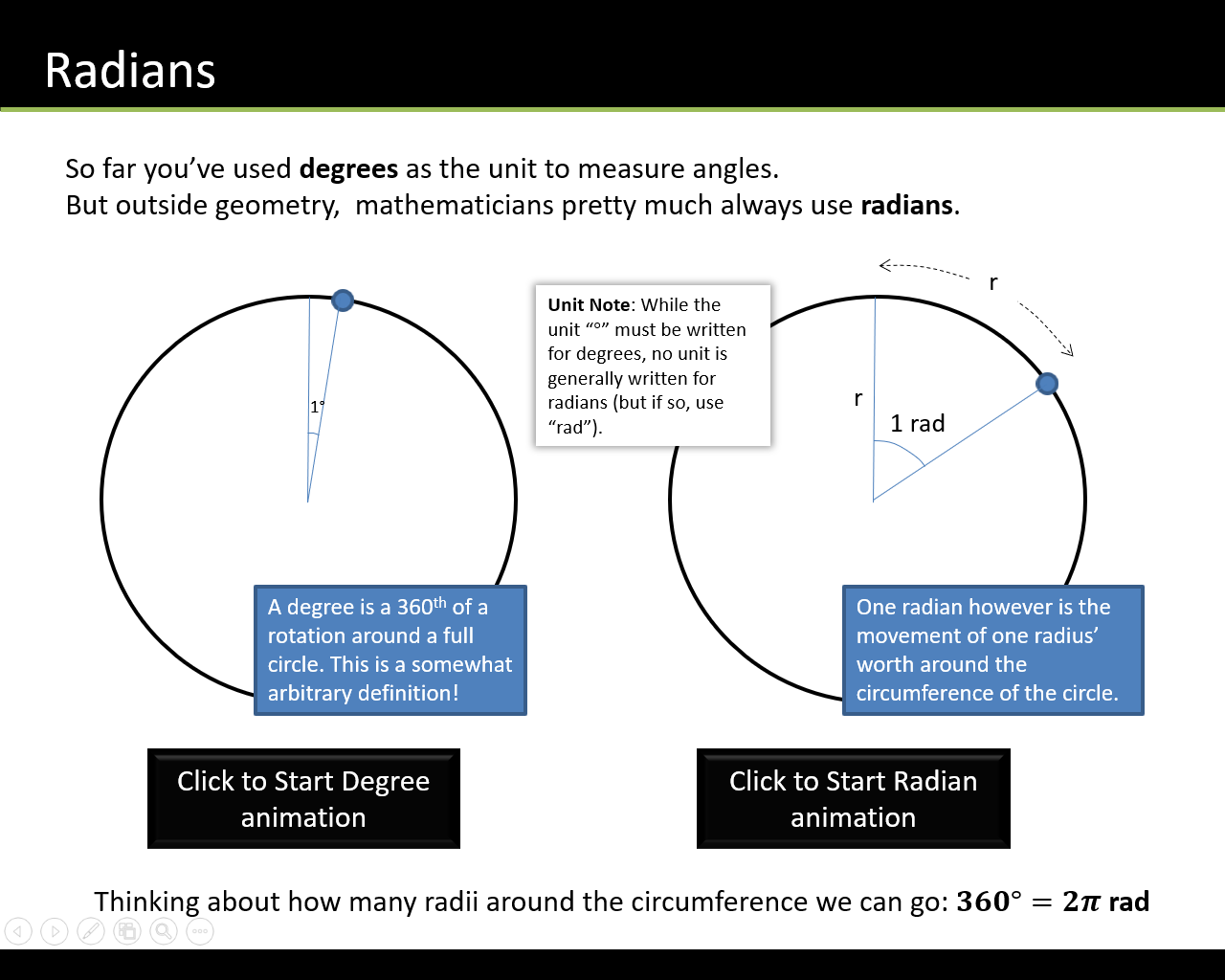
**2**: Find arc length and sector area (when using radians)

**3**: Solve trig equations in radians.

**4**: Small angle approximations



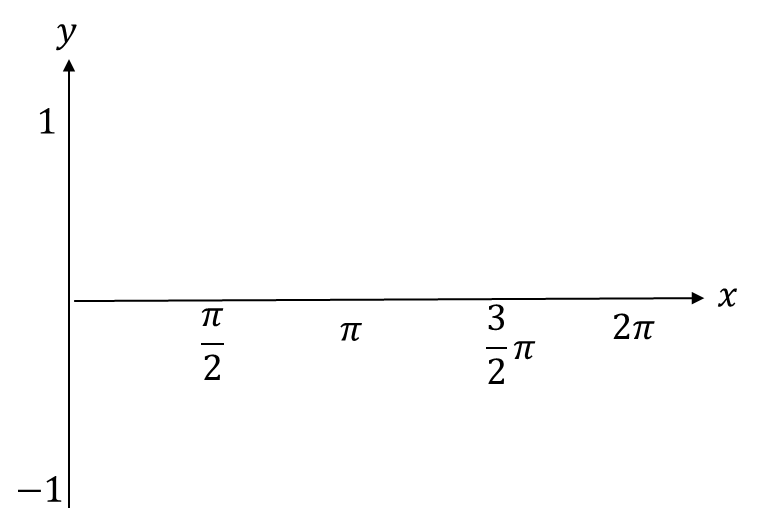
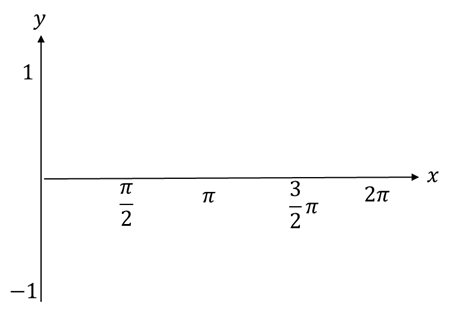
Radians



Converting between radians and degrees

It is useful to remember the standard angle conversions….

Graph Sketching with Radians



Test Your Understanding

Sketch the graph of for

Sin, cos, tan of angles in radians

Reminder of laws from Year 1:

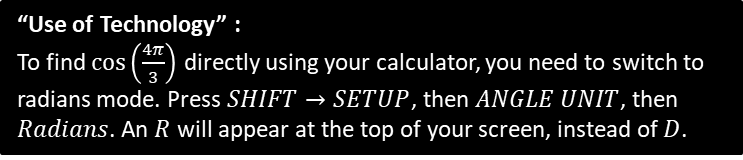
* repeat every but every

In terms of radians:

* repeat every \_\_\_\_\_\_\_ but every \_\_\_\_\_.

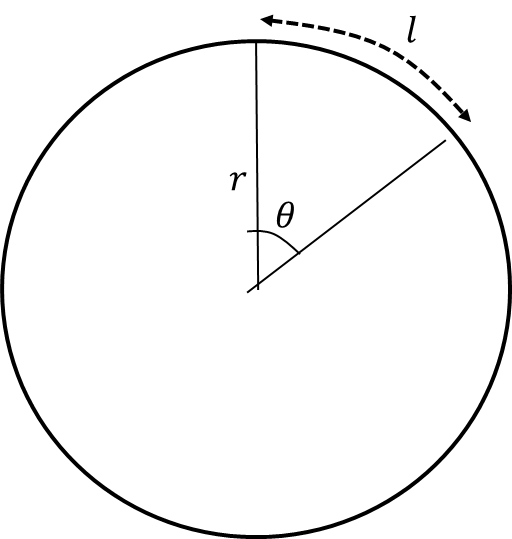
To find sin/cos/tan of a ‘**common**’ angle in radians without using a calculator, it is easiest to just **convert to degrees first**.

Examples



Page 116/118 Ex 5a/ 5b

Arc length

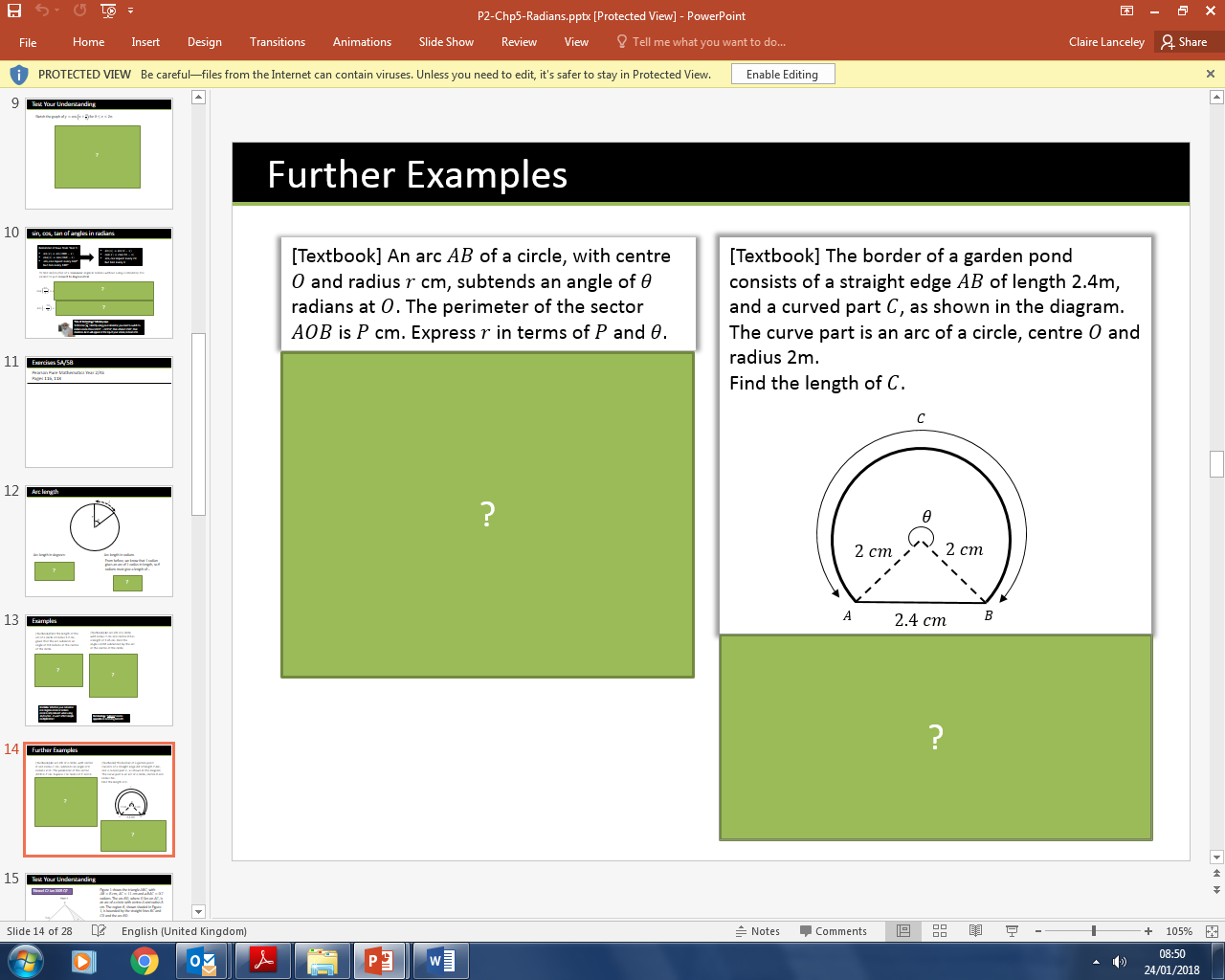


Arc length in degrees =

Arc length in radians =

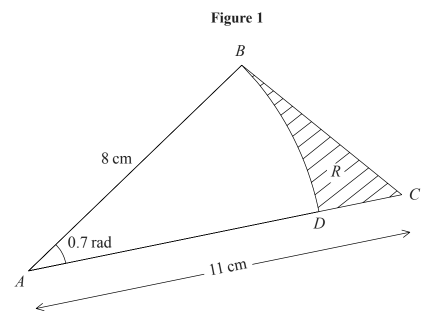
Examples

1. Find the length of the arc of a circle of radius 5.2 cm, given that the arc subtends an angle of 0.8 radians at the centre of the circle.
2. An arc of a circle with radius 7 cm and centre has a length of 2.45 cm. Find the angle subtended by the arc at the centre of the circle
3. An arc of a circle, with centre and radius cm, subtends an angle of radians at . The perimeter of the sector is cm. Express in terms of and
4. The border of a garden pond consists of a straight edge of length 2.4m, and a curved part , as shown in the diagram. The curve part is an arc of a circle, centre and radius 2m.

Find the length of .

Test Your Understanding

Figure 1 shows the triangle , with , and radians. The arc , where lies on , is an arc of a circle with centre and radius 8 cm. The region , shown shaded in Figure 1, is bounded by the straight lines and and the arc .

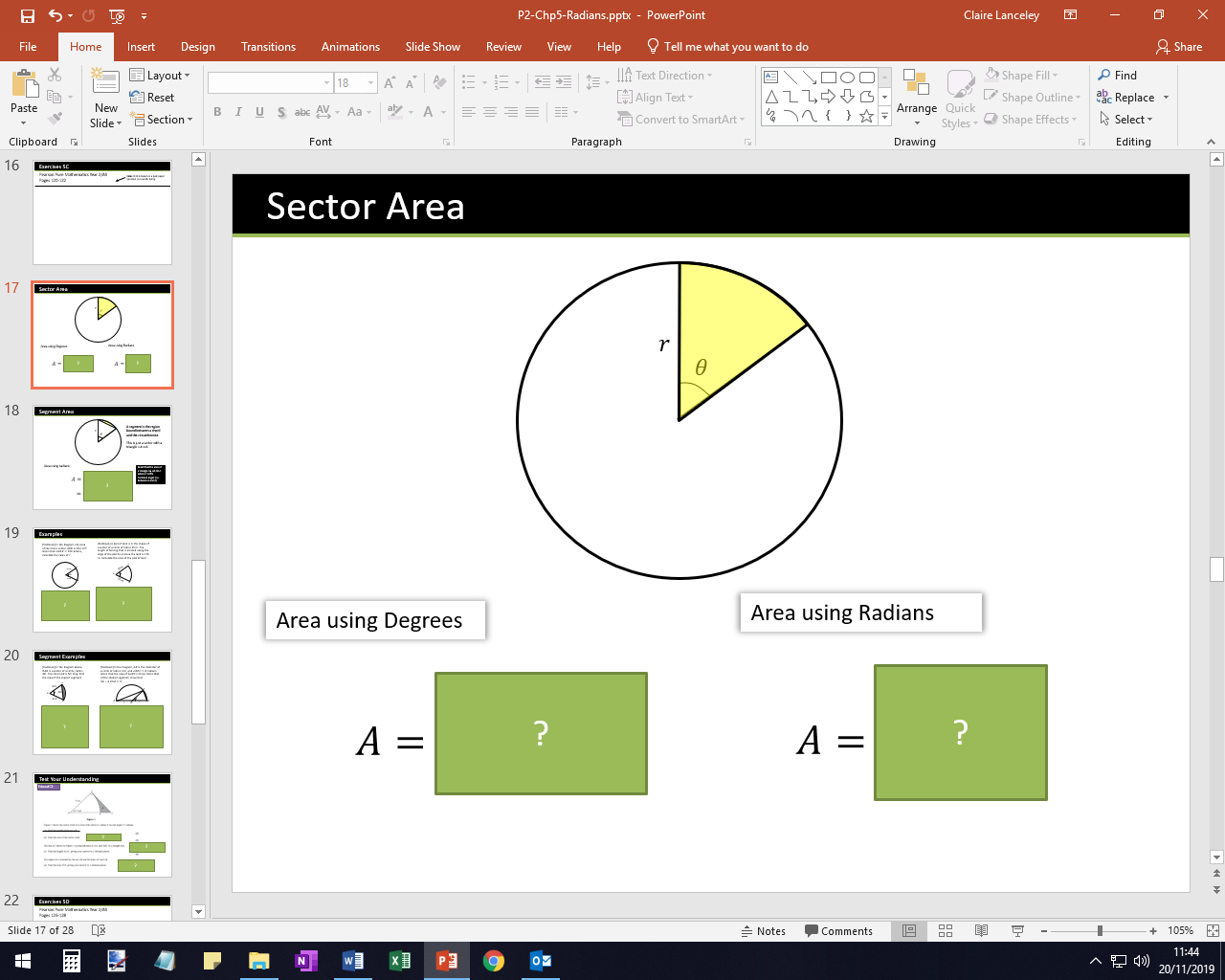
Find

(a) The length of the arc .

(b) The perimeter of , giving your answer to 3 significant figures.

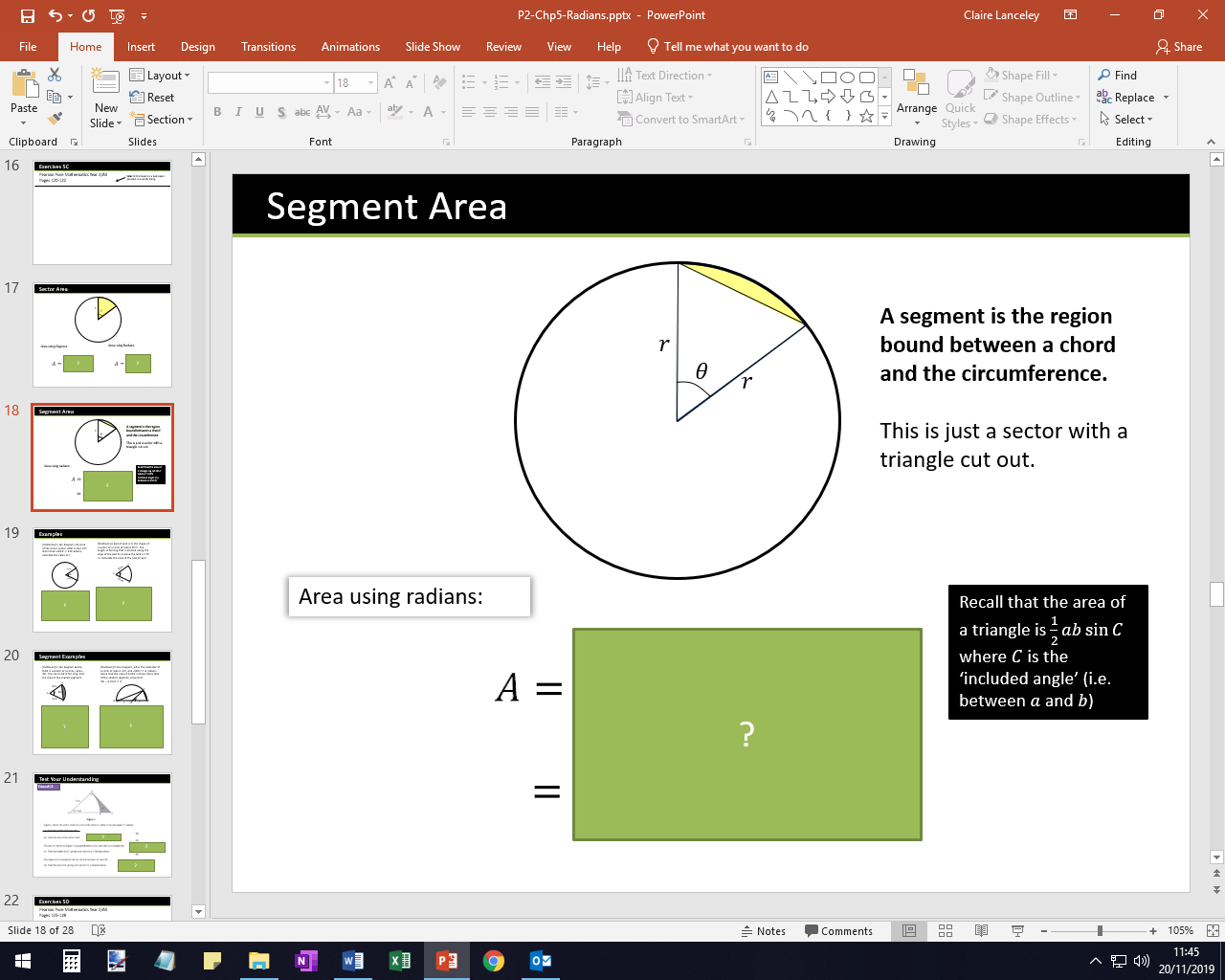
Ex 5C Page 120

Sector Area

 Area using Degrees =

Area using Radians =

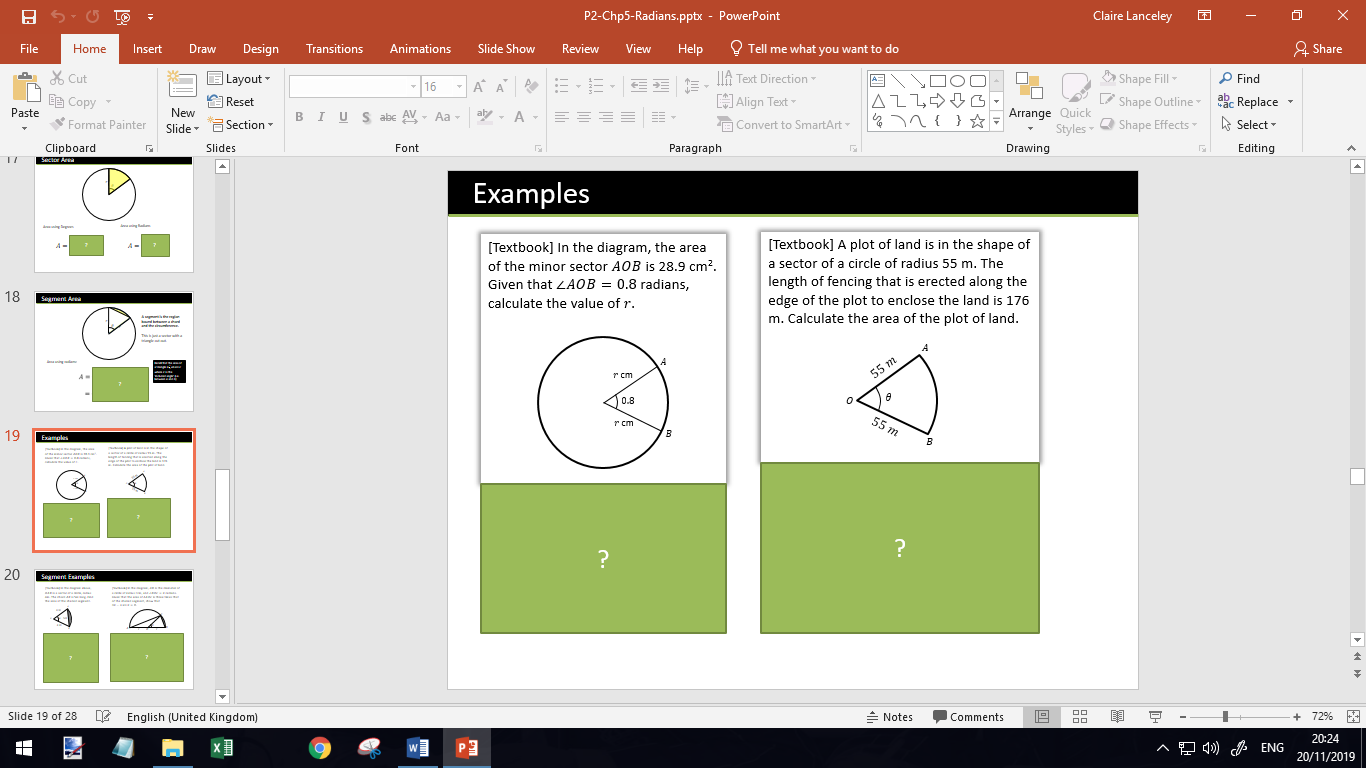
Segment Area

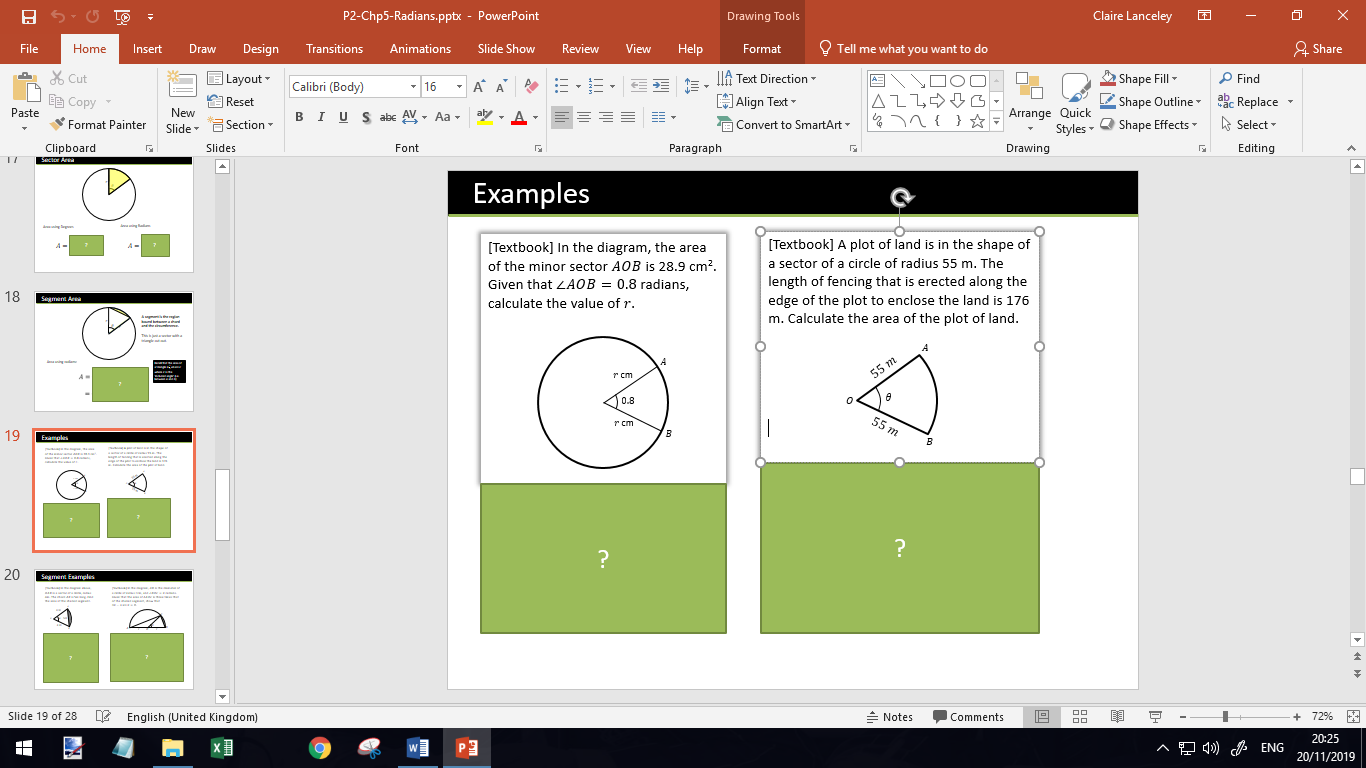


Recall that the area of a triangle is where is the ‘included angle’ (i.e. between and )

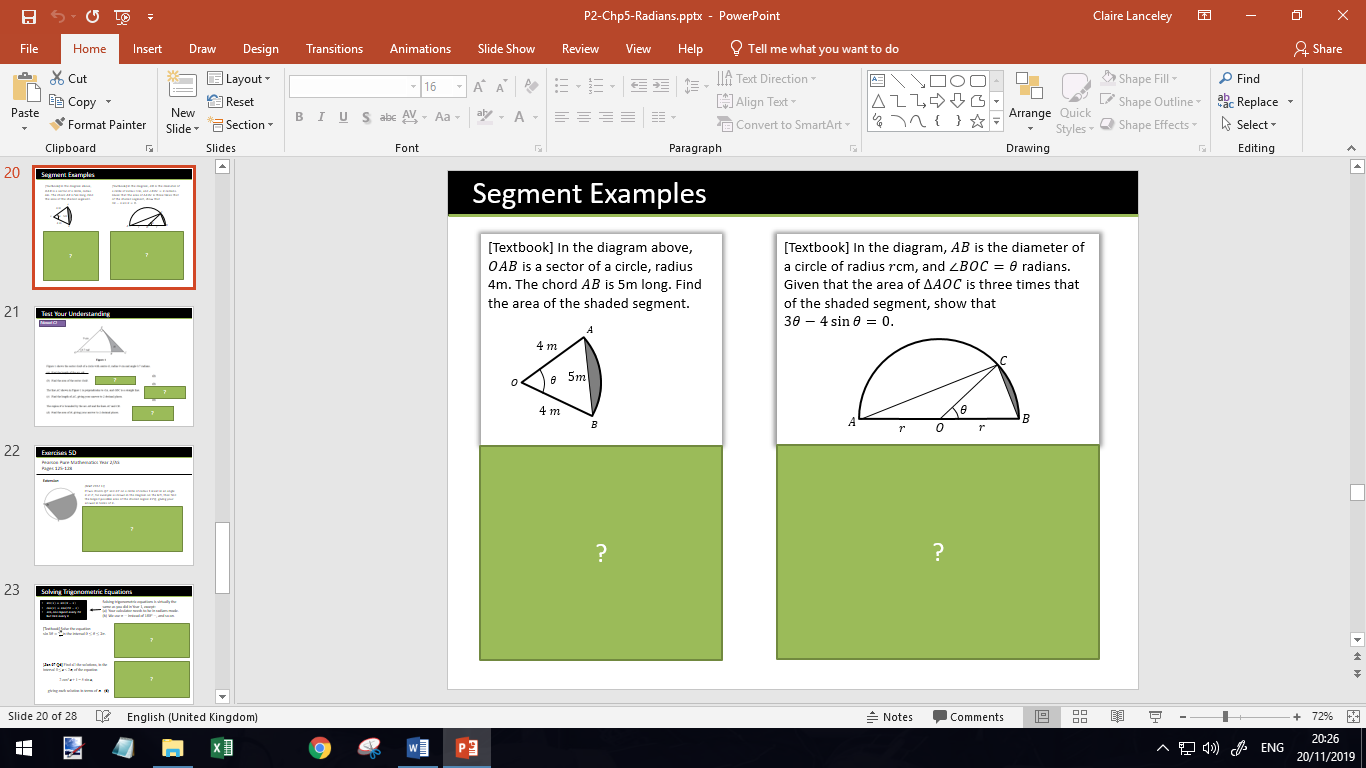
Area using radians:

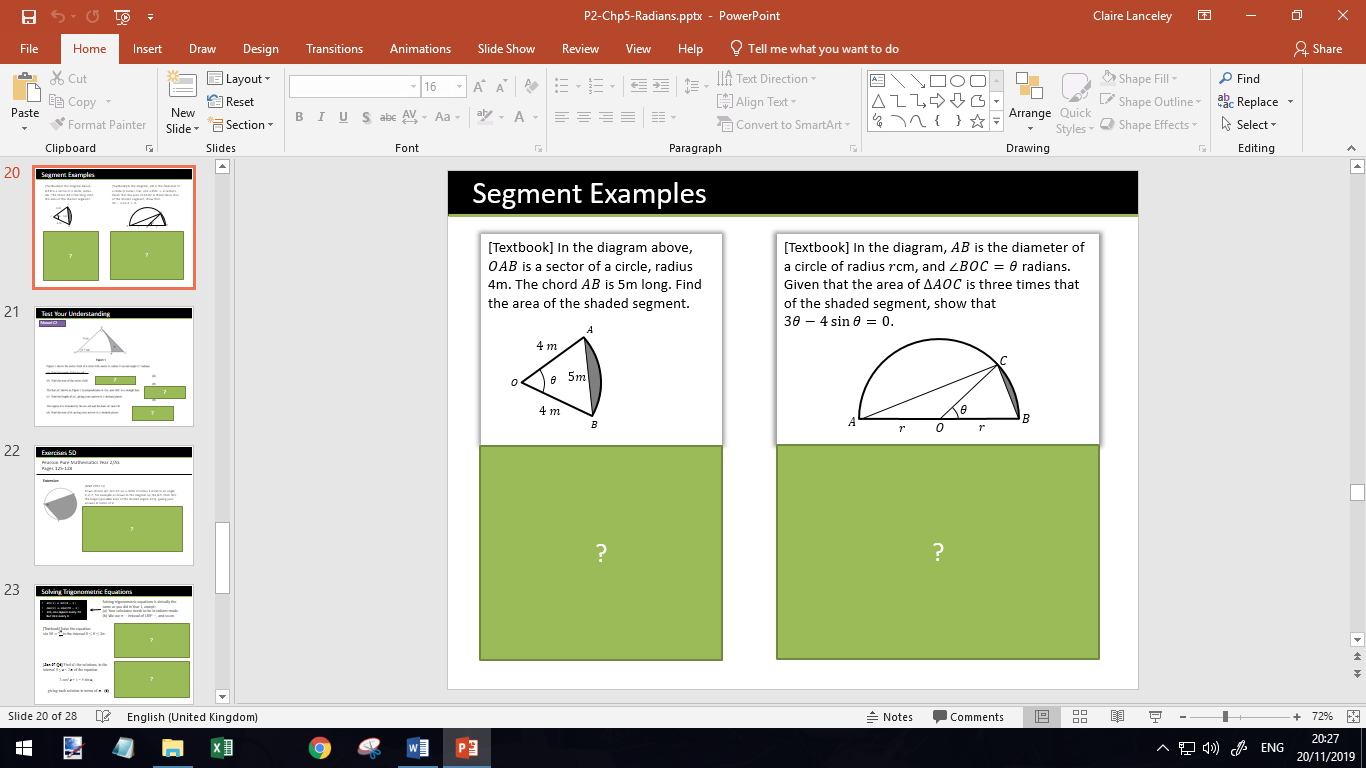
Examples

1. In the diagram, the area of the minor sector is 28.9 cm2. Given that radians, calculate the value of .
2. A plot of land is in the shape of a sector of a circle of radius 55 m. The length of fencing that is erected along the edge of the plot to enclose the land is 176 m. Calculate the area of the plot of land.

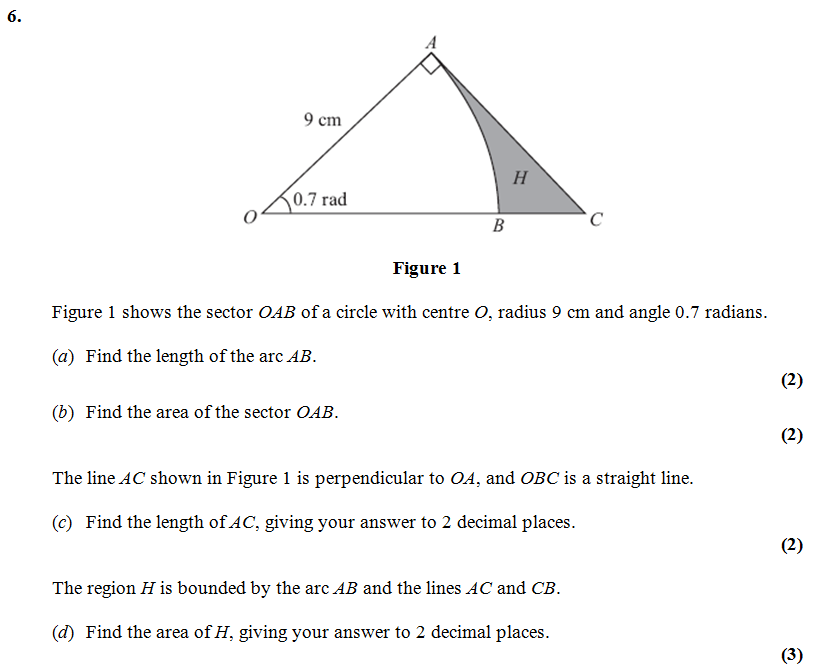


1. In the diagram above, is a sector of a circle, radius 4m. The chord is 5m long. Find the area of the shaded segment.



1. In the diagram, is the diameter of a circle of radius cm, and radians. Given that the area of is three times that of the shaded segment, show that   
   .

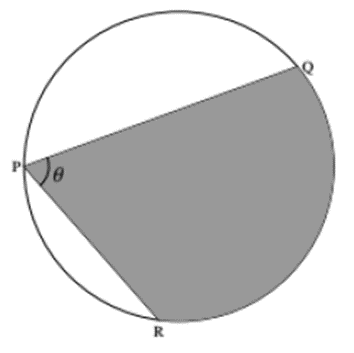
Test Your Understanding



Extension

*[MAT 2012 1J]*

If two chords and on a circle of radius 1 meet in an angle at , for example as drawn in the diagram on the left, then find the largest possible area of the shaded region , giving your answer in terms of .



Ex 5D Page 125 - 128

Solving Trigonometric Equations

Solving trigonometric equations is virtually the same as you did in Year 1, except:

1. Your calculator needs to be in radians mode.
2. We use instead of , and so on.

Remember

* repeat every but every

Example

Solve the equation   
 in the interval .

Test Your Understanding

**[Jan 07 Q6]**

Find all the solutions, in the interval 0 ≤ *x* < 2*π*, of the equation

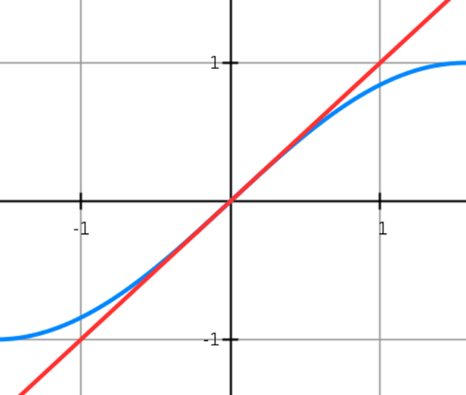
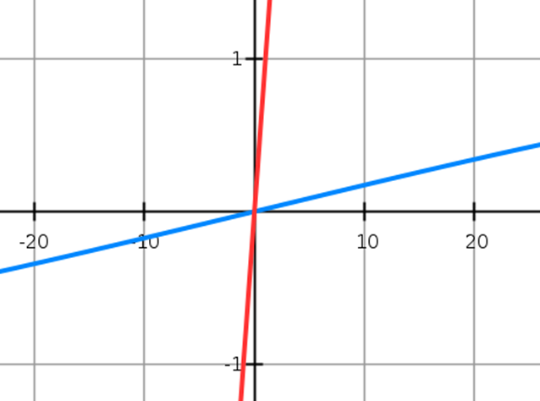
2 cos2 *x* + 1 = 5 sin *x*, giving each solution in terms of *π*. **(6)**

Extension

[MAT 2010 1C] In the range , the equation has how many solutions?

Ex 5E Pg 131

Small Angle Approximations

Example

When is small, find the approximate value of:



Example

a) Show that, when is small,

b) Hence state the approximate value of   
 for small values of .

Ex 5F Pg 134