Lower 6 Chapter 14

Exponentials and logarithms

Chapter Overview

1. Sketch exponential graphs.
2. Use and interpret models that use exponential functions.
3. Be able to differentiate .
4. Understand the log function and use laws of logs.
5. Use logarithms to estimate values of constants in non-linear models.



**Contrasting exponential graphs**

On the same axes sketch ,

On the same axes sketch and

Graph Transformations

Sketch

Exercise 14A Pg 313-314

AD Page 11

**Differentiating**

If , where is a constant, then

Different with respect to .

Different with respect to .

Different with respect to .

**More Graph Transformations**

Sketch

Sketch

Sketch

Sketch

Exercise 14B Pg 316-317

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**Exponential Modelling**

There are two key features of exponential functions which make them suitable for **population growth**:

1. **gets times bigger each time increases by 1. (Because )**With population growth, we typically have a fixed percentage increase each year. So suppose the growth was 10% a year, and we used the equivalent decimal multiplier, 1.1, as . Then , where is the number of years, would get 1.1 times bigger each year.
2. **The rate of increase is proportional to the size of the population at a given moment.**
This makes sense: The 10% increase of a population will be twice as large if the population itself is twice as large.

**Example**

[Textbook] The density of a pesticide in a given section of field, mg/m2, can be modelled by the equation

where is the time in days since the pesticide was first applied.

a. Use this model to estimate the density of pesticide after 15 days.

b. Interpret the meaning of the value 160 in this model.

c. Show that , where is a constant, and state the value of .

d. Interpret the significance of the sign of your answer in part (c).

e. Sketch the graph of against .

Exercise 14C Pg 318-319

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**Logarithms**

 (“said log base of ”) is equivalent to .
 The log function outputs the **missing power**.

**Examples**

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With your calculator…

Exercise 14D Pg 320-321

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**Laws of logs**

Three main laws:

Special cases:

Not in syllabus (but in MAT/PAT):

**Examples**

Write as a single logarithm:

1.
2.

Write in terms of , and

1.
2.
3.
4.

**Solving equations with logs**

Solve the equation



Exercise 14E Pg 323-324

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**Solving equations with exponential terms**

Solve

Solve

Solve

Solve the equation , giving your answer to 3sf.

Solve , giving your answer to 3dp.

Solve , giving your answer in exact form.

Solve , giving your answer to 3dp.

Exercise 14F Pg 325

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**Natural logarithms**

The inverse of is

Solve

Solve

Solve

Solve

Solve

Solve

Solve giving your answer as an exact value.

Exercise 14G Pg 327-8

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**Graphs for Exponential Data**

Turning non-linear graphs into linear ones

[Textbook] The graph represents the growth of a population of bacteria, , over hours. The graph has a gradient of 0.6 and meets the vertical axis at as shown.

A scientist suggests that this growth can be modelled by the equation , where and are constants to be found.

1. Write down an equation for the line.
2. Using your answer to part (a) or otherwise, find the values of and , giving them to 3 sf where necessary.

Interpret the meaning of the constant in this model.





Dr Frost’s wants to predict his number of Twitter followers (@DrFrostMaths) years from the start 2015. He predicts that his followers will increase exponentially according to the model , where are constants that he wishes to find.

He records his followers at certain times. Here is the data:

**Years after 2015**: 0.7 1.3 2.2

**Followers** : 2353 3673 7162

1. Draw a table giving values of and (to 3dp).
2. A line of best fit is drawn for the data in your new table, and it happens to go through the first data point above (where ) and last (where ).
Determine the equation of this line of best fit. (The -intercept is 3.147)
3. Hence, determine the values of and in the model.
4. Estimate how many followers Dr Frost will have at the start of 2020 (when ).

Exercise 14H Pg 331-333

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