

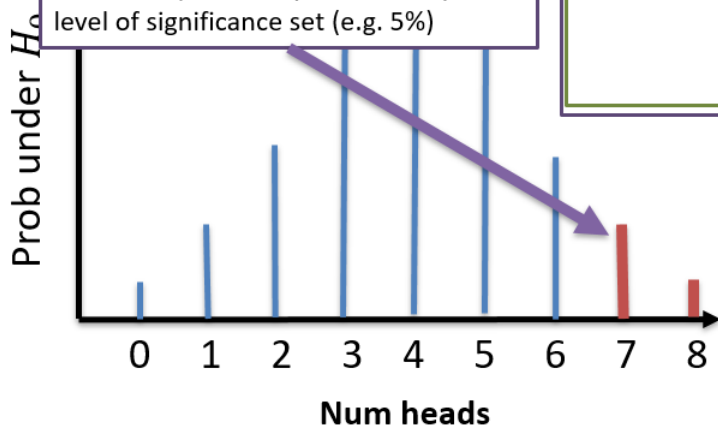
Critical Regions and Values

John wants to see whether a coin is unbiased or whether **it is biased towards coming down heads**. He tosses the coin 8 times and counts the number of times X , it lands head uppermost. **What values would lead to John's hypothesis being rejected?**

As before, we're interested how likely a given outcome is likely to happen 'just by chance' under the null hypothesis (i.e. when the coin is not biased).

However, there are values which collectively form a range of 'extreme values' where it would be unlikely that the coin would be unbiased. Their combined probability is limited by the level of significance set (e.g. 5%)

The probability of getting exactly 5 heads is only 22%, which is more likely to not happen than to happen. If we saw this number of heads, why would it not be sensible to think the coin is biased?



Critical Regions and Values


John wants to see whether a coin is unbiased or whether **it is biased towards coming down heads**. He tosses the coin 8 times and counts the number of times X , it lands head uppermost. **What values would lead to John's hypothesis being rejected, if the significance level was 5%?**


What's the probability that we would see **6 heads**, or an **even more extreme value**? Is this sufficiently unlikely to support John's claim that the coin is biased?

What's the probability that we would see **7 heads**, or an **even more extreme value**?

C.D.F. Binomial table:
 $p = 0.5, n = 8$

x	$P(X \leq x)$
0	0.0039
1	0.0352
2	0.1445
3	0.3633
4	0.6367
5	0.8555
6	0.9648
7	0.9961

 The **critical region** is the range of values of the test statistic that would lead to you rejecting H_0

 The value(s) on the boundary of the critical region are called **critical value(s)**.

Critical value:

C.D.F. Binomial table:
 $p = 0.5, n = 8$

x	$P(X \leq x)$
0	0.0039
1	0.0352
2	0.1445
3	0.3633
4	0.6367
5	0.8555
6	0.9648
7	0.9961
8	1

Quick fire Critical Regions

Determine the critical region when we throw a coin where we're trying to establish if there's the specified bias, given the specified number of throws, when the level of significance is 5%.

Coin thrown 5 times. Trying to establish if biased towards heads.

$$p = 0.5, n = 5$$

x	$P(X \leq x)$
0	0.0312
1	0.1875
2	0.5000
3	0.8125
4	0.9688

Critical region:

Coin thrown 10 times. Trying to establish if biased towards heads.

$$p = 0.5, n = 10$$

x	$P(X \leq x)$
0	0.0010
1	0.0107
2	0.0547
...	...
7	0.9453
8	0.9893
9	0.9990

Critical region:

Coin thrown 10 times. Trying to establish if biased towards tails.

$$p = 0.5, n = 10$$

x	$P(X \leq x)$
0	0.0010
1	0.0107
2	0.0547
...	...
7	0.9453
8	0.9893
9	0.9990

Critical region:

For Reminder: At the positive tail, use the value AFTER the first that exceeds 95% (100 - 5).

At the negative tail, we just use the first value that goes under the significance level.