Chapter 7 - Statistics

Hypothesis Testing

Chapter Overview

1. Hypothesis Testing
2. Finding Critical Values
3. One-Tailed Tests
4. Two-Tailed Tests

Topics	What students need to learn:		
	Content		Guidance
5 **Statistical hypothesis testing** *continued*	5.2	**Conduct a statistical hypothesis test for the proportion in the binomial distribution and interpret the results in context.**	
		Understand that a sample is being used to make an inference about the population. **and** **appreciate that the significance level is the probability of incorrectly rejecting the null hypothesis.**	**Hypotheses should be expressed in terms of the population parameter p** **A formal understanding of Type I errors is not expected.**
	5.3	Conduct a statistical hypothesis test for the mean of a Normal distribution with known, given or assumed variance and interpret the results in context.	Students should know that: If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$ and that a test for μ can be carried out using: $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1^2)$. No proofs required. Hypotheses should be stated in terms of the population mean μ. Knowledge of the Central Limit Theorem or other large sample approximations is not required.

1. Hypothesis Testing

What is Hypothesis Testing?

Vocabulary:

> **Hypothesis** = statement about the value of a population parameter
> **Test Statistic** = the result of the experiment, or the statistic that is calculated from the example
> **Null Hypothesis, H_0** = the hypothesis you assume is correct
> **Alternative Hypothesis, H_1** = tells you about the parameter if your assumption is shown to be wrong

Example

10% of the world's population are left-handed. On my holiday to Hawaii, I want to establish if the proportion of left-handed people in Hawaii is greater than the world average. I have a table of 20 people as my sample. I need to ensure any result I get is **statistically significant**.

1) What is the hypothesis?

2) What is the population parameter?

In my sample of 20 people in Hawaii, I find that 5 are left-handed.
3) Suggest a null hypothesis (the hypothesis I assume is correct)

4) Suggest an alternative hypothesis (what happens to my parameter if my assumption is wrong)

5) What is the test statistic?

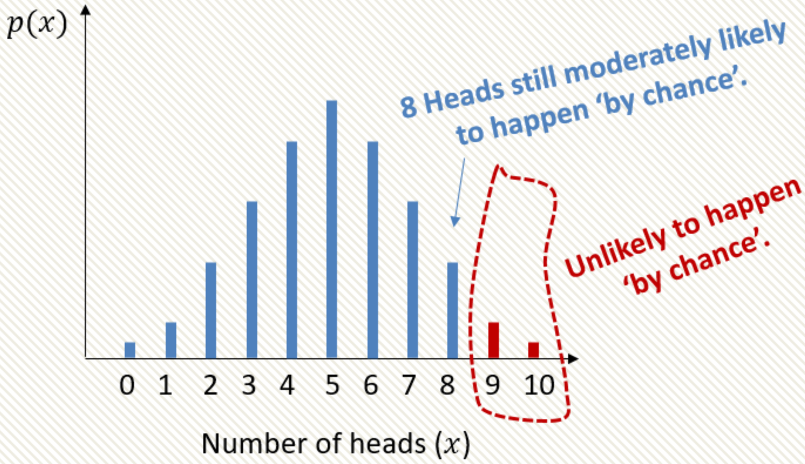
What is Hypothesis Testing?

Hypothesis testing in a nutshell then is:

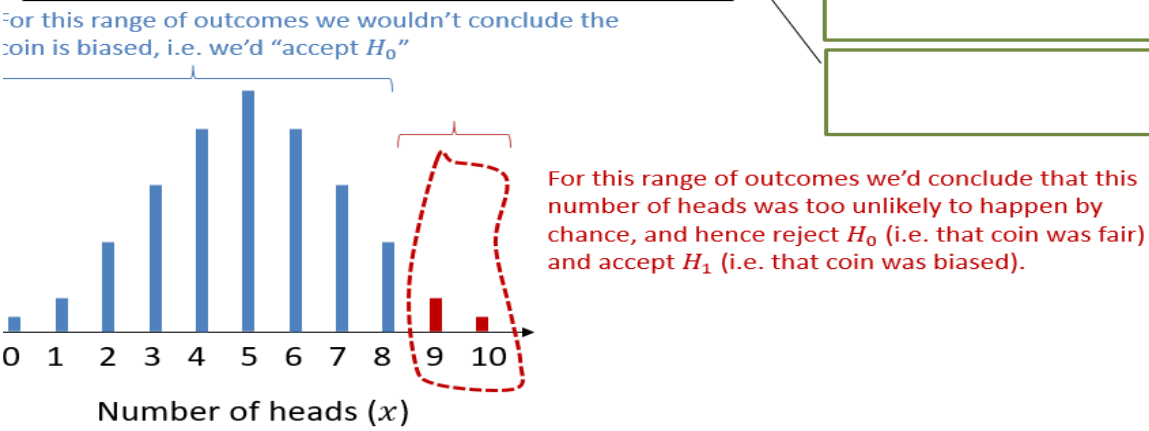
1. We have some hypothesis we wish to see if true (proportion of left-handed people in Hawaii is more than global average), so…
2. We collect some sample data (giving us our test statistic) and…
3. If that data is sufficiently unlikely to have emerged 'just by chance', then we conclude that our (alternative) hypothesis is correct.

I throw a coin 10 times. For what numbers of heads might you conclude that the coin is biased towards heads? Why?

Our intuition is a large number of heads of low number of heads, far away from the 'expected' number of 5 heads out of 10. There is because the probability of this number of heads occurring 'by chance' (i.e. if the coin was in fact fair) is low.



In this context…



Null Hypothesis and Alternative Hypothesis

[Textbook] John wants to see whether a coin is unbiased or whether **it is biased towards coming down heads**. He tosses the coin 8 times and counts the number of times X, it lands head uppermost.

We said that our two hypotheses are about the population parameter.

Suppose p is the probability of a coin landing heads.

Null hypothesis:

Alternative hypothesis:

Under the **null hypothesis** H_0, we **assume that the population parameter is correct**, in this case, that it is a normal coin and the probability of Heads is 0.5

Under the **alternative hypothesis** H_1, there has been an underlying change in the population parameter, in this case that the coin is actually biased towards Heads

The latter is known as a '**one-tailed test**' because we're saying the coin is biased one way or the other (i.e. $p > 0.5$ or $p < 0.5$).
But we could also have had the hypothesis 'the coin is biased (either way)', i.e. $p \neq 0.5$. This is known as a **two-tailed test**.

Further Example

[Textbook] An election candidate believes she has the support of 40% of the residents in a particular town. A researcher wants to test, at the 5% significance level, whether the candidate is over-estimating her support. The researcher asks 20 people whether they support the candidate or not. 3 people say they do.

a) Write down a suitable test statistic.

b) Write down two suitable hypotheses.

c) Explain the condition under which the null hypothesis would be rejected.

a

For a hypothesis test involving the binomial distribution, the test statistic is always the **count of successes**.

b

The alternative hypothesis is that the candidate is **overestimating** her support, so we're interested where **less than 40%** support them (more than 40% would not undermine the candidate's claim).

c

This is the hard bit!
We always calculate the probability of seeing this outcome **or more extreme** (in this case, 'more extreme' meaning even fewer the 3 people, because this takes us even further from the expected number of people out of the 20 (i.e. 8) who would support them.
The "$p = 0.4$" bit is because, as discussed before, we calculate the probability of seeing the observed outcome of 3 people (or more extreme) if it occurred **purely by chance** (the null hypothesis), i.e. if the candidate **did** have 40% support.