

Leftie Example

Let's simplify the problem by using just 3 people:

The probability a randomly chosen person is left-handed is 0.1. If there is a group of 3 people, what is the probability that:

- a) All 3 are left-handed.
- b) 0 are left-handed.
- c) 1 person is left-handed.
- d) 2 people are left-handed.

d

a

b

c


Let's try to generalise!

If there were x 'lefties' out of 3, then we can see, using the examples, that the probability of a single matching outcome is $0.1^x \times 0.9^{3-x}$. How many rows did we have each time? In a sequence of three L's and R's, there are "3 choose x ", i.e. $\binom{3}{x}$ ways of choosing x of the 3 letters to be L's. Therefore the probability of x out of 3 people being left handed is:

$$\binom{3}{x} 0.1^x 0.9^{3-x}$$



The Binomial Distribution

 You can model a random variable X with a binomial distribution $B(n, p)$ if

- there are a fixed number of trials, n ,
- there are two possible outcomes: 'success' and 'failure',
- there is a fixed probability of success, p
- the trials are independent of each other

← In our example, 'success' was 'leftie'.

If $X \sim B(n, p)$ then:

$$P(X = r) = \binom{n}{r} p^r (1 - p)^{n-r}$$

← r is the number of successes out of n .

← " \sim " means "has the distribution"

On a table of 8 people, 6 people are left handed.

- a) Suggest a suitable model for a random variable X : the number of left-handed people in a group of 8, where the probability of being left-handed is 0.1.
- b) Find the probability 6 people are left handed.
- c) Suggest why the chosen model may not have been appropriate.

a

b

c

In general, choosing a well-known model, such as a Binomial distribution, makes certain **simplifying assumptions**. Such assumptions simplifies the maths involved, but potentially at the expense of not adequately modelling the situation.



The random variable $X \sim B\left(12, \frac{1}{6}\right)$. Find:

- a) $P(X = 2)$
- b) $P(X = 9)$
- c) $P(X \leq 1)$

A company claims that a quarter of the bolts sent to them are faulty. To test this claim the number of faulty bolts in a random sample of 50 is recorded.

(a) Give two reasons why a binomial distribution may be a suitable model for the number of faulty bolts in the sample. (2)

Test Your Understanding

1 $X \sim B(6, 0.2)$
What is $P(X = 2)$?

What is $P(X \geq 5)$?

2 I have a bag of 2 red and 8 white balls. X represents the number of red balls I chose after 5 selections (with replacement).

How is X distributed?

Determine the probability that I chose 3 red balls.

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An awkward Tiffin boy ventures into Tiffin Girls. He asks 20 girls out on the date. The probability that each girl says yes is 0.3.

Determine the probability that he will end up with:

- a) Less than 6 girls on his next date.
- b) At least 9 girls on his next date.



The boy considers the evening a success if he dated at least 9 girls that evening.

He repeats this process across 5 evenings.

- c) Calculate the probability that he had at least 4 successful evenings.

(Note: You won't be able to use your table for (c) as p is not a nice round number – calculate prob directly)