Hypothesis Testing on the Sample Mean

Examples

[Textbook] A certain company sells fruit juice in cartons. The amount of juice in a carton has a normal distribution with a standard deviation of 3ml.
The company claims that the mean amount of juice per carton, μ, is 60ml. A trading inspector has received complaints that the company is overstating the mean amount of juice per carton and wishes to investigate this complaint. The trading inspector takes a random sample of 16 cartons and finds that the mean amount of juice per carton is 59.1ml.

Using a 5% level of significance, and stating your hypotheses clearly, test whether or not there is evidence to uphold this complaint.

Finding the critical region

[Textbook] A machine products bolts of diameter D where D has a normal distribution with mean 0.580 cm and standard deviation 0.015 cm. The machine is serviced and after the service a random sample of 50 bolts from the next production run is taken to see if the mean diameter of the bolts has changed from 0.580 cm. The distribution of the diameters of bolts after the service is still normal with a standard deviation of 0.015 cm.

(a) Find, at the 1% level, the critical region for this test, stating your hypotheses clearly.
The mean diameter of the sample of 50 bolts is calculated to be 0.587 cm.
(b) Comment on this observation in light of the critical region.

Test Your Understanding

Roastie's Coffee is sold in packets with a stated weight of 250 g. A supermarket manager claims that the mean weight of the packets is less than the stated weight. She weighs a random sample of 90 packets from their stock and finds that their weights have a mean of 248 g and a standard deviation of 5.4 g.

(a) Using a 5% level of significance, test whether or not the manager's claim is justified. State your hypotheses clearly.

(5)

Conditional Probabilities

This is not in the textbook. But given the recent Chapter 2 on Conditional Probabilities and the fact that the type of question below occurred frequently in S1 papers, it seems worthwhile to cover!

The time, in minutes, taken to fly from London to Malaga has a normal distribution with mean 150 minutes and standard deviation 10 minutes.

The time, X minutes, taken to fly from London to another city has a normal distribution with mean μ minutes.

Given that $P(X < \mu - 15) = 0.35$

(c) find $P(X > \mu + 15 \mid X > \mu - 15)$.

(3)

Test Your Understanding

The length of time, L hours, that a phone will work before it needs charging is normally distributed with a mean of 100 hours and a standard deviation of 15 hours.

(a) Find $P(L > 127)$.

(3)

Alice is about to go on a 6 hour journey. Given that it is 127 hours since Alice last charged her phone,

(c) find the probability that her phone will not need charging before her journey is completed.

(4)

One Last Toughie…

IQ is distributed as $X \sim N(100, 15^2)$.
A person is considered to be 'pretty smart' if their IQ is at least 130.
Determine the median IQ of 'pretty smart' people.