Standard Normal Distribution

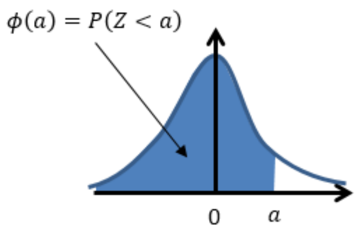
✎ Z is the

If again we use IQ distributed as $X \sim N(100, 15^2)$ then:

IQ	Z
100	
130	
85	
165	
62.5	

$$Z = \frac{X - \mu}{\sigma}$$
and $Z \sim N(0, 1^2)$. Z is known as a **standard** normal distribution.

Standard Normal Distribution



✎ $\Phi(a) = P(Z < a)$ is the cumulative distribution for the standard normal distribution. The values of $\Phi(a)$ can be found in a z-table.

This is a traditional z-table in the old A Level syllabus (but also found elsewhere). You no longer get given this and are expected to use your calculator.

THE NORMAL DISTRIBUTION FUNCTION

The function tabulated below is $\Phi(z)$, defined as $\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}t^2} \, dt$.

z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$
0.00	0.5000	0.50	0.6915	1.00	0.8413	1.50	0.9332	2.00	0.9772
0.01	0.5040	0.51	0.6950	1.01	0.8438	1.51	0.9345	2.02	0.9783
0.02	0.5080	0.52	0.6985	1.02	0.8461	1.52	0.9357	2.04	0.9793
0.03	0.5120	0.53	0.7019	1.03	0.8485	1.53	0.9370	2.06	0.9803
0.04	0.5160	0.54	0.7054	1.04	0.8508	1.54	0.9382	2.08	0.9812
0.05	0.5199	0.55	0.7088	1.05	0.8531	1.55	0.9394	2.10	0.9821
0.06	0.5239	0.56	0.7123	1.06	0.8554	1.56	0.9406	2.12	0.9830
0.07	0.5279	0.57	0.7157	1.07	0.8577	1.57	0.9418	2.14	0.9838
0.08	0.5319	0.58	0.7190	1.08	0.8599	1.58	0.9429	2.16	0.9846
0.09	0.5359	0.59	0.7224	1.09	0.8621	1.59	0.9441	2.18	0.9854
0.10	0.5398	0.60	0.7257	1.10	0.8643	1.60	0.9452	2.20	0.9861

This is from the new formula booklet. This is sometimes known as a 'reverse z-table', because you're looking up the z-value for a probability. Beware: p here it the probability of **exceeding** z rather than being up to z. Let's use it…

Percentage Points of The Normal Distribution

The values z in the table are those which a random variable $Z \sim N(0, 1)$ exceeds with probability p; that is, $P(Z > z) = 1 - \Phi(z) = p$.

p	z	p	z
0.5000	0.0000	0.0500	1.6449
0.4000	0.2533	0.0250	1.9600
0.3000	0.5244	0.0100	2.3263
0.2000	0.8416	0.0050	2.5758
0.1500	1.0364	0.0010	3.0902
0.1000	1.2816	0.0005	3.2905

Examples

[Textbook] The random variable $X \sim N(50, 4^2)$. Write in terms of $\Phi(z)$ for some value of z.
(a) $P(X < 53)$ (b) $P(X \geq 55)$

a

b

[Textbook] The systolic blood pressure of an adult population, S mmHg, is modelled as a normal distribution with mean 127 and standard deviation 16. A medical research wants to study adults with blood pressures higher than the 95th percentile. Find the minimum blood pressure for an adult included in her study.

p	z	p	z
0.5000	0.0000	0.0500	1.6449
0.4000	0.2533	0.0250	1.9600
0.3000	0.5244	0.0100	2.3263
0.2000	0.8416	0.0050	2.5758
0.1500	1.0364	0.0010	3.0902
0.1000	1.2816	0.0005	3.2905

Further Examples

(a) Determine $P(Z > -1.3)$
(b) Determine $P(-2 < Z < 1)$
(c) Determine the a such that $P(Z > a) = 0.7$
(d) Determine the a such that $P(-a < Z < a) = 0.6$

p	z	p	z
0.5000	0.0000	0.0500	1.6449
0.4000	0.2533	0.0250	1.9600
0.3000	0.5244	0.0100	2.3263
0.2000	0.8416	0.0050	2.5758
0.1500	1.0364	0.0010	3.0902
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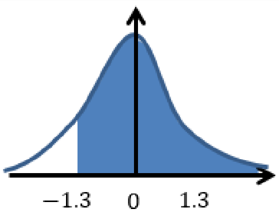
a



Test Your Understanding

1 IQ is distributed with mean 100 and standard deviation 15. Using an appropriate table, determine the IQ corresponding to the

(a) top 10% of people.

(b) bottom 20% of people.

p	z	p	z
0.5000	0.0000	0.0500	1.6449
0.4000	0.2533	0.0250	1.9600
0.3000	0.5244	0.0100	2.3263
0.2000	0.8416	0.0050	2.5758
0.1500	1.0364	0.0010	3.0902
0.1000	1.2816	0.0005	3.2905

a

b

2 If $X \sim N(100, 15^2)$, determine, in terms of Φ:

(a) $P(X > 115)$

(b) $P(77.5 < X < 112)$

a

b

3 Find the a such that:

(a) $P(-a < Z < a) = 0.2$

(b) $P(0 < Z < a) = 0.35$

a

b