Chapter 3

The Normal Distribution

Chapter Overview

1:: Characteristics of the Normal Distribution

What shape is it? What parameters does it have?

2:: Finding probabilities on a standard normal curve.

"Given that IQ is distributed as $X \sim N(100, 15^2)$, determine the probability that a randomly chosen person has an IQ above 130."

3:: Finding unknown means/standard deviations.

In Wales, 30% of people have a height above 1.6m. Given the mean height is 1.4m and heights are normally distributed, determine the standard deviation of heights.

:: Binomial → Normal Approximations

How would I approximate $X \sim B(10, 0.4)$ using a Normal distribution? Under what conditions can we make such an approximation?

5:: Hypothesis Testing

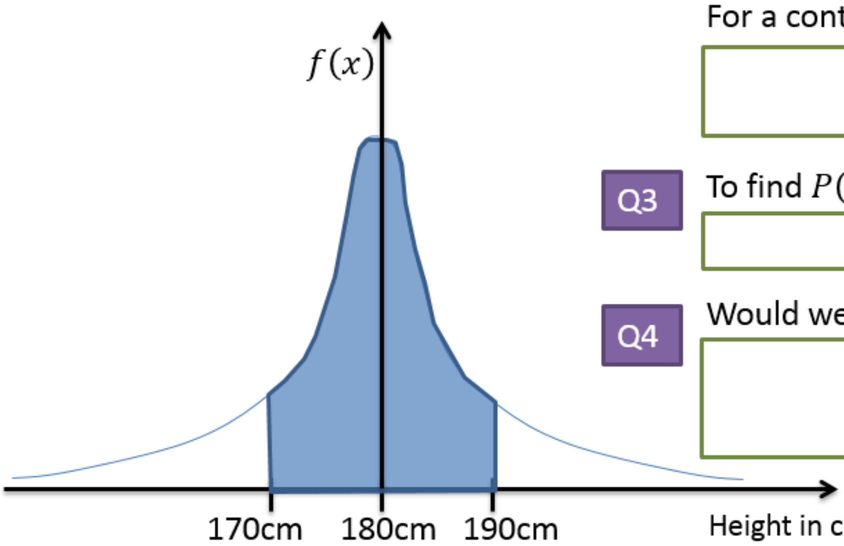
Specification

4.2	Understand and use the Normal distribution as a model; find probabilities using the Normal distribution	The notation $X \sim N(\mu, \sigma^2)$ may be used. Knowledge of the shape and the symmetry of the distribution is required. Knowledge of the probability density function is not required. Derivation of the mean, variance and cumulative distribution function is not required. Questions may involve the solution of simultaneous equations. Students will be expected to use their calculator to find probabilities connected with the normal distribution.
	Link to histograms, mean, standard deviation, points of inflection	Students should know that the points of inflection on the normal curve are at $x = \mu \pm \sigma$. The derivation of this result is not expected.
	and the binomial distribution.	Students should know that when n is large and p is close to 0.5 the distribution $B(n, p)$ can be approximated by $N(np, np[1-p])$ The application of a continuity correction is expected.

What does it look like?

The following shows what the probability distribution might look like for a random variable X, if X is the height of a randomly chosen person.

Normal Distribution Q & A



Q1 For a Normal Distribution to be used, the variable has to be:

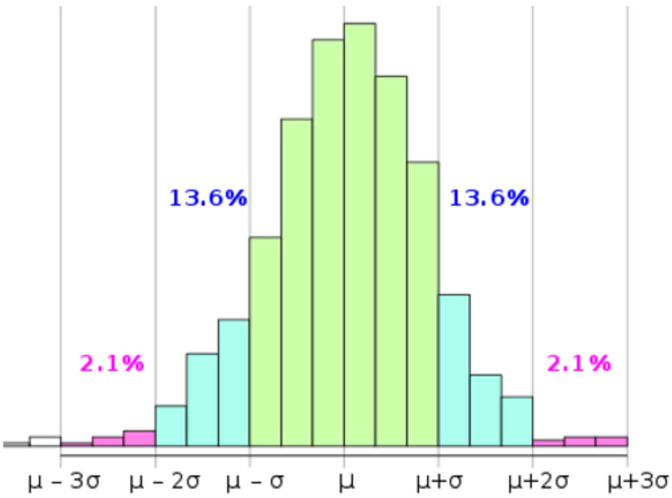
Q2 With a discrete variable, all the probabilities had to add up to 1.
For a continuous variable, similarly:

Q3 To find $P(170 < X < 190)$, we could:

Q4 Would we ever want to find $P(X = 200)$ say?

The 68-95-99.7 rule

<u>You need to memorise this!</u>



The histogram above is for a quantity which is approximately normally distributed.

$\approx 68\%$

$\approx 95\%$

$\approx 99.7\%$

For practical purposes we consider all data to lie within $\mu \pm 5\sigma$

Examples

[Textbook] The diameters of a rivet produced by a particular machine, X mm, is modelled as $X \sim N(8, 0.2^2)$. Find:

a) $P(X > 8)$

b) $P(7.8 < X < 8.2)$

Test Your Understanding

IQ ("Intelligence Quotient") for a given population is, by definition, distributed using $X \sim N(100, 15^2)$. Find:

a) $P(70 < X < 130)$
b) $P(X > 115)$