3A The Normal Distribution

Key Values:

1. The diameters of a rivet produced by a particular machine, X mm, is modelled as $X \sim N(8, 0.2^2)$. Find:

a) $P(X > 8)$

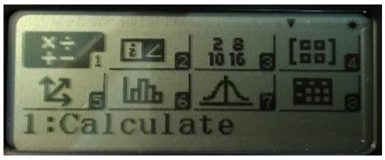
b) $P(7.6 < X < 8.4)$

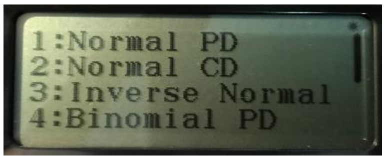
c) $P(X > 7.8)$

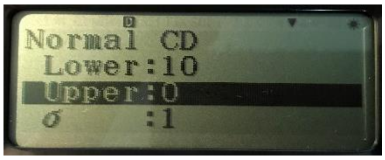
3B Finding Probabilities

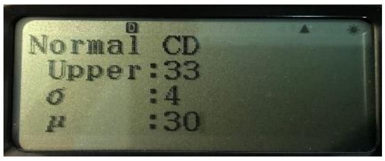
1. Given that $X \sim N(30, 4^2)$, find:

a) $P(X < 33)$









b) $P(X \geq 24)$

c) $P(33.5 < X < 38.2)$

d) $P(X < 27 \text{ } or \text{ } X > 32)$

2. An IQ test is applied to a population of adults. The scores, X, on the test are found to be normally distributed with $X \sim N(100, 15^2)$. Adults scoring more that 140 on the test are classified as 'genius'.

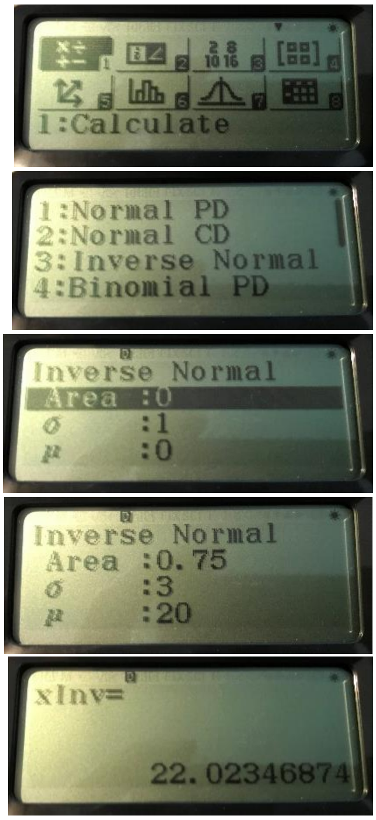
a) Find the probability that an adult chosen at random achieves a 'genius' classification

b) Twenty adults take the test. Find the probability that two or more and classified as 'genius'

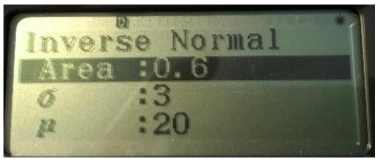
<u>3C Finding Values From Probabilities (Inverse Function)</u>

1. Given that $X \sim N(20, 3^2)$, find, to two decimal places, the values of a such that:

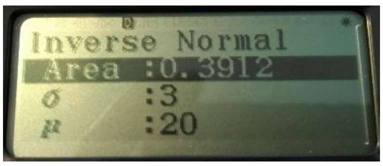
a) $P(X < a) = 0.75$



b) $P(X > a) = 0.4$



c) $P(16 < X < a) = 0.3$



2. Plates made using a particular manufacturing process have a diameter, $D\ cm$, which can be modelled using a normal distribution, $D \sim N(20, 1.5^2)$.

a) Given that 60% of plates are less than $x\ cm$, find the value of x.

b) Find the interquartile range of the plate diameters

3D The Normal Normal Distribution (Z Distribution)

1. The random variable $X \sim N(50, 4^2)$. Write in terms of $\Phi(z)$ for some value z:

a) $P(X < 53)$

b) $P(X \geq 55)$

2. The systolic blood pressure (pressure when the heart beats) of an adult population, $S\ mmHg$, is modelled as a normal distribution with mean 127 and standard deviation 16.

   A medical researcher wants to study adults with blood pressures higher than the 95th percentile. Find the minimum blood pressure needed for an adult to be included in her survey

3E Finding the Mean or Standard Deviation

1. The random variable $X \sim N(\mu, 3^2)$. Given that $P(X > 20) = 0.2$, find the value of μ.

2. A machine makes metal sheets with width $X\ cm$, modelled as a normal distribution such that $X \sim N(50, \sigma^2)$.

a) Given that $P(X < 46) = 0.2119$, find the value of σ.

b) Find the 90th percentile of the widths

3. The random variable $X \sim N(\mu, \sigma^2)$. Given that $P(X > 35) = 0.025$ and $P(X < 15) = 0.1469$, find the values of μ and σ.

3F Approximating from the Binomial Distribution

1. A biased coin has $P(Head) = 0.53$. The coin is tossed 100 times and the number of heads, X, is recorded.

a) Write down a binomial model for X

b) Explain why X can be approximated using a normal distribution

c) Find the values of μ and σ in this approximation

2. The binomial random variable $X \sim B(150, 0.48)$ is approximated by the normal random variable $Y \sim N(72, 6.12^2)$.

a) Use this approximation to find $P(X \leq 70)$

b) Also use the approximation to find $P(80 \leq X < 90)$

3. For a particular type of flower bulb, 55% will produce yellow flowers. A random sample of 80 bulbs is planted.

   Calculate the percentage error incurred when using a normal approximation to estimate the probability that there are <u>exactly</u> 50 yellow flowers.

3G Hypothesis Testing

1. A company sells fruit juice in cartons. The amount of juice in a carton has a normal distribution with a standard deviation of 3ml.

   The company claims that the mean amount of juice per carton, μ, is 60ml. A trading inspector has received complaints that the company is overstating the amount of juice per carton and wishes to investigate this complaint.

   The inspector takes a random sample of 16 cartons and finds that the mean amount of juice per carton is 59.1ml.

   Using a 5% significance level, and stating your hypotheses clearly, test whether or not there is sufficient evidence to uphold the complaints.

2. A machine produces bolts of diameter D where D has a normal distribution with mean 0.580cm and standard deviation 0.015cm.

   This machine is serviced and after the service a random sample of 50 bolts from the next production is taken to see if the mean diameter of the bolts has changed from 0.580cm. The distribution of the bolts after the service is still normal with a standard deviation of 0.015cm.

a) Find, at the 1% level, the critical region for this test, stating your hypotheses clearly

b) The mean diameter of a sample of 50 bolts is found to be 0.587mm. Comment on this in light of the critical region