Graphs for Exponential Data

Turning non-linear graphs into linear ones

Case 1: Polynomial → Linear

Suppose our original model was a polynomial one*:

$$y = ax^n$$

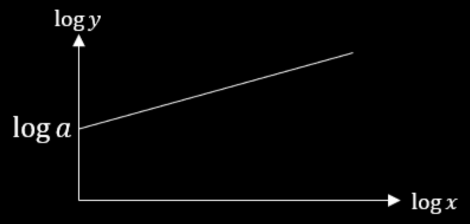
Then taking logs of both sides:

$$\log y = \log ax^n$$
$$\log y = \log a + n \log x$$

We can compare this against a straight line:

$$Y = mX + c$$

✎ If $y = ax^n$, then the graph of $\log y$ against $\log x$ will be a straight line wih gradient n and vertical intercept $\log a$.



* We could also allow non-integer n; the term would then not strictly be polynomial, but we'd still say the function had "polynomial growth".

Case 2: Exponential → Linear

Suppose our original model was an exponential one:

$$y = ab^x$$

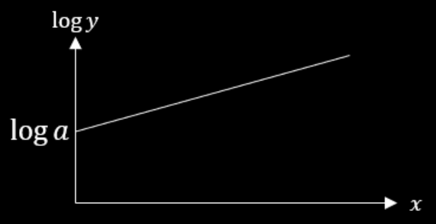
Then taking logs of both sides:

$$\log y = \log ab^x$$
$$\log y = \log a + x \log b$$

Again we can compare this against a straight line:

$$Y = mX + c$$

✎ If $y = ab^x$, then the graph of $\log y$ against x will be a straight line with gradient $\log b$ and vertical intercept $\log a$.



The key difference compared to Case 1 is that we're **only logging the y values** (e.g. number of transistors), not the x values (e.g. years elapsed). **Note that you <u>do not need to memorise</u> the contents of these boxes and we will work out from scratch each time…**

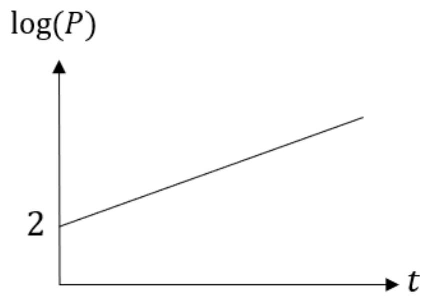
In summary, logging the y-axis **turns an exponential graph into a linear one**. Logging **both** the x and y-axis turns a polynomial graph into a linear one.

[Textbook] The graph represents the growth of a population of bacteria, P, over t hours. The graph has a gradient of 0.6 and meets the vertical axis at $(0,2)$ as shown.

A scientist suggests that this growth can be modelled by the equation $P = ab^t$, where a and b are constants to be found.

a. Write down an equation for the line.
b. Using your answer to part (a) or otherwise, find the values of a and b, giving them to 3 sf where necessary.

Interpret the meaning of the constant a in this model.



[Textbook] The table below gives the rank (by size) and population of the UK's largest cities and districts (London is number 1 but has been excluded as an outlier).

City	B'ham	Leeds	Glasgow	Sheffield	Bradford
Rank, R	2	3	4	5	6
Population, P	1 000 000	730 000	620 000	530 000	480 000

The relationship between the rank and population can be modelled by the formula:
$P = aR^n$ where a and n are constants.

Textbook Error: They use $R = aP^n$ but then plot $\log P$ against $\log R$.

a) Draw a table giving values of $\log R$ and $\log P$ to 2dp.
b) Plot a graph of $\log R$ against $\log P$ using the values from your table and draw the line of best fit.
c) Use your graph to estimate the values of a and n to two significant figures.

Dr Frost's wants to predict his number of Twitter followers P (@DrFrostMaths) t years from the start 2015. He predicts that his followers will increase exponentially according to the model $P = ab^t$, where a, b are constants that he wishes to find.
He records his followers at certain times. Here is the data:

Years t after 2015:	0.7	1.3	2.2
Followers P:	2353	3673	7162

a) Draw a table giving values of t and $\log P$ (to 3dp).
b) A line of best fit is drawn for the data in your new table, and it happens to go through the first data point above (where $t = 0.7$) and last (where $t = 2.2$).
Determine the equation of this line of best fit. (The y-intercept is 3.147)
c) Hence, determine the values of a and b in the model.
d) Estimate how many followers Dr Frost will have at the start of 2020 (when $t = 5$).

Exercise 14H Pg 331-333