## Lower 6 Chapter 14

## Exponentials and logarithms

## Chapter Overview

1. Sketch exponential graphs.
2. Use and interpret models that use exponential functions.
3. Be able to differentiate $e^{k x}$.
4. Understand the log function and use laws of logs.
5. Use logarithms to estimate values of constants in nonlinear models.

| 6 <br> Exponentials and logarithms | 6.1 | Know and use the function $a^{x}$ and its graph, where $a$ is positive. <br> Know and use the function $\mathrm{e}^{x}$ and its graph | Understand the difference in shape between $a<1$ and $a>1$ |
| :---: | :---: | :---: | :---: |
|  | 6.2 | Know that the gradient of $\mathrm{e}^{k x}$ is equal to $k \mathrm{e}^{k x}$ and hence understand why the exponential model is suitable in many applications. | Realise that when the rate of change is proportional to the $y$ value, an exponential model should be used. |
| 6 <br> Exponentials and logarithms continued | 6.3 | Know and use the definition of $\log _{a} x$ as the inverse of $a^{x}$, where $a$ is positive and $x \geqslant 0$ <br> Know and use the function $\ln x$ and its graph <br> Know and use $\ln x$ as the inverse function of $\mathrm{e}^{x}$ | $a \neq 1$ <br> Solution of equations of the form $\mathrm{e}^{a x+b}=p$ and $\ln (a x+b)=q$ is expected. |
|  | 6.4 | Understand and use the laws of logarithms: $\begin{aligned} \log _{a} x+\log _{a} y & =\log _{a}(x y) \\ \log _{a} x-\log _{a} y & =\log _{a}\left(\frac{x}{y}\right) \end{aligned}$ $k \log _{a} x=\log _{a} x^{k}$ <br> (including, for example, $\left.k=-1 \text { and } k=-\frac{1}{2}\right)$ | Includes $\log _{a} a=1$ |
|  | 6.5 | Solve equations of the form $a^{x}=b$ | Students may use the change of base formula. Questions may be of the form, for example, $2^{3 x-1}=3$ |
|  | 6.6 | Use logarithmic graphs to estimate parameters in relationships of the form $y=a x^{n}$ and $y=k b^{x}$, given data for $x$ and $y$ | Plot $\log y$ against $\log x$ and obtain a straight line where the intercept is $\log a$ and the gradient is $n$ <br> Plot $\log y$ against $x$ and obtain a straight line where the intercept is $\log k$ and the gradient is $\log b$ |
|  | 6.7 | Understand and use exponential growth and decay; use in modelling (examples may include the use of e in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as a model for population growth); consideration of limitations and refinements of exponential models. | Students may be asked to find the constants used in a model. <br> They need to be familiar with terms such as initial, meaning when $t=0$. <br> They may need to explore the behaviour for large values of $t$ or to consider whether the range of values predicted is appropriate. <br> Consideration of a second improved model may be required. |

## Contrasting exponential graphs

On the same axes sketch $y=3^{x}, y=2^{x}, y=1.5^{x}$

On the same axes sketch $y=2^{x}$ and $y=\left(\frac{1}{2}\right)^{x}$

Graph Transformations
Sketch $y=2^{x+3}$

