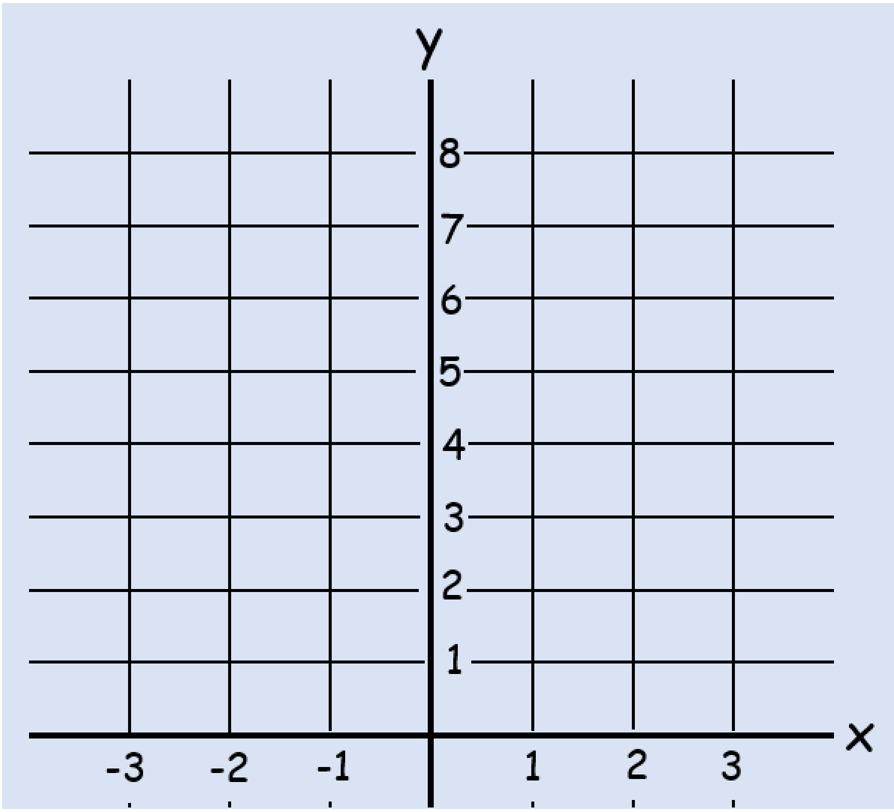
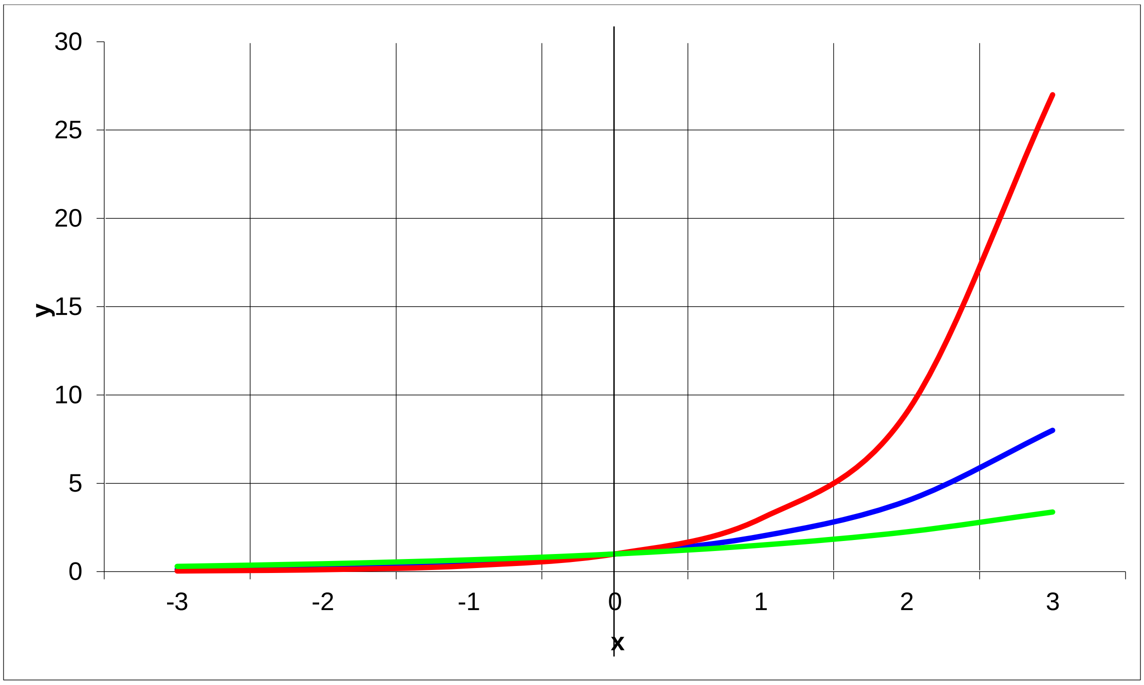
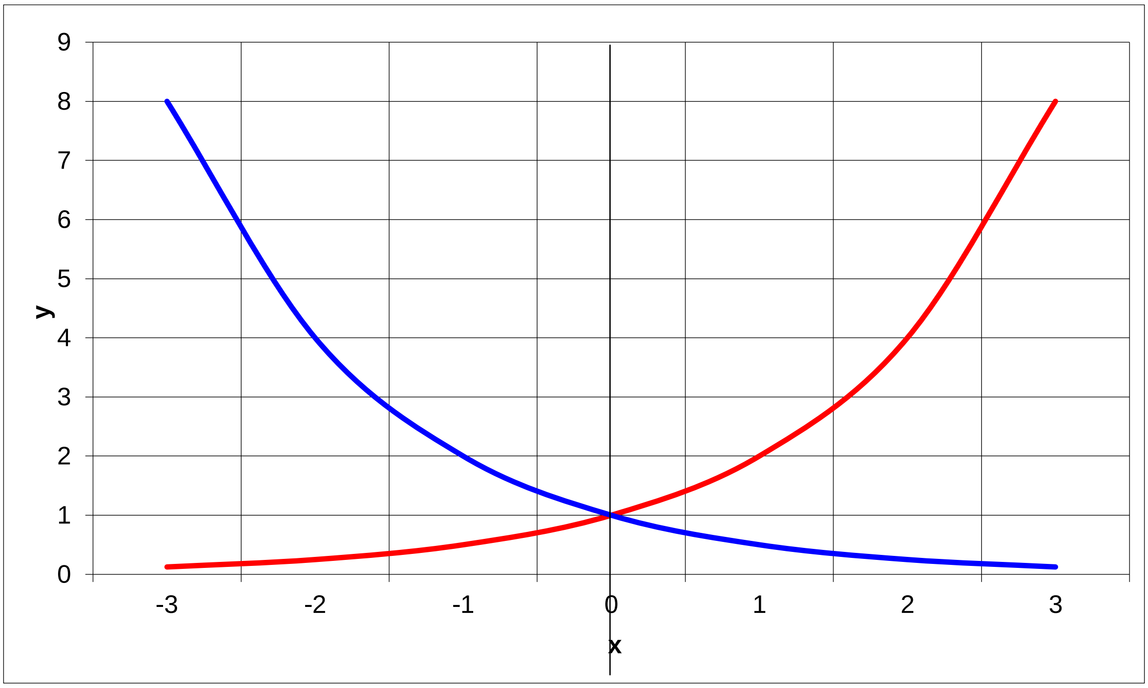
14A Drawing Exponential Graphs

1) Draw the graph of $y = 2^x$

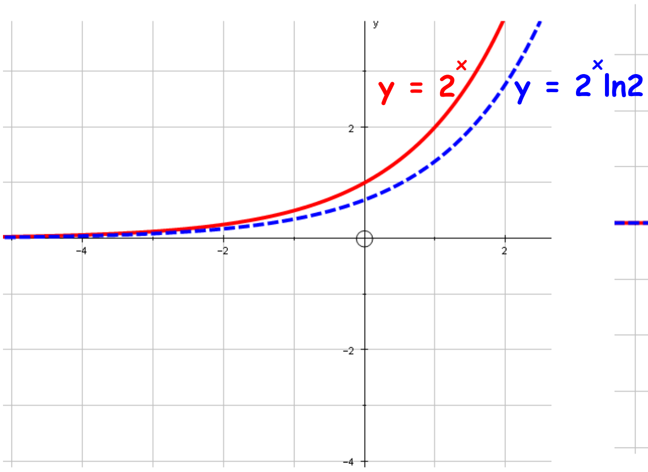
x	-3	-2	-1	0	1	2	3
y							

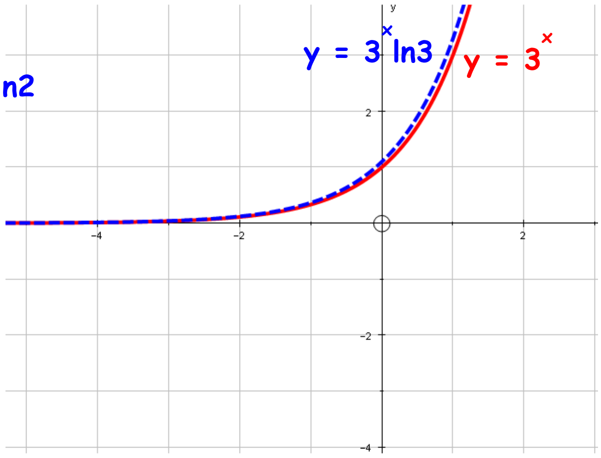


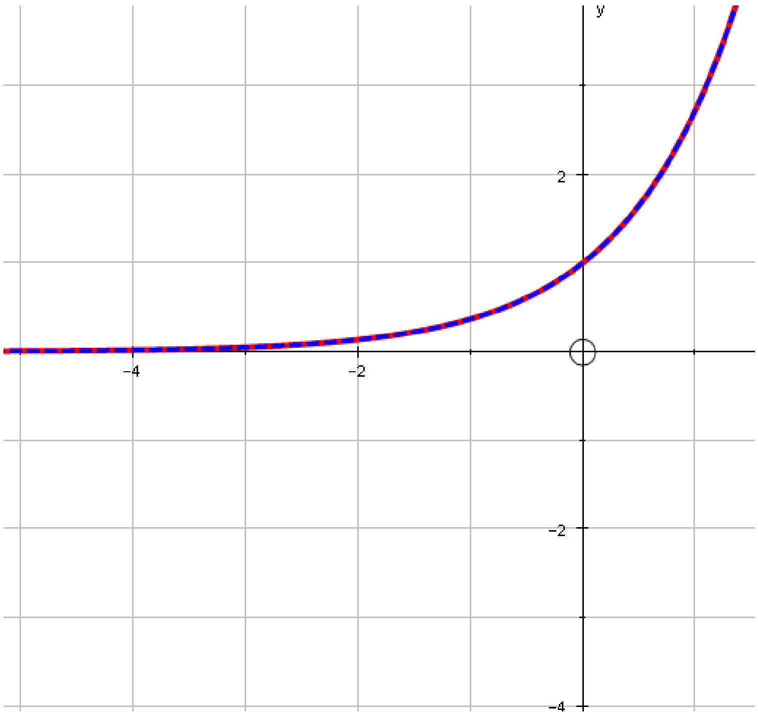

30
25
20
15
10
5
0
y
-3
-2
-1
0
1
2
3
x


9
8
7
6
5
4
3
2
1
0
y
-3
-2
-1
0
1
2
3
x

14B Euler's Number 'e'







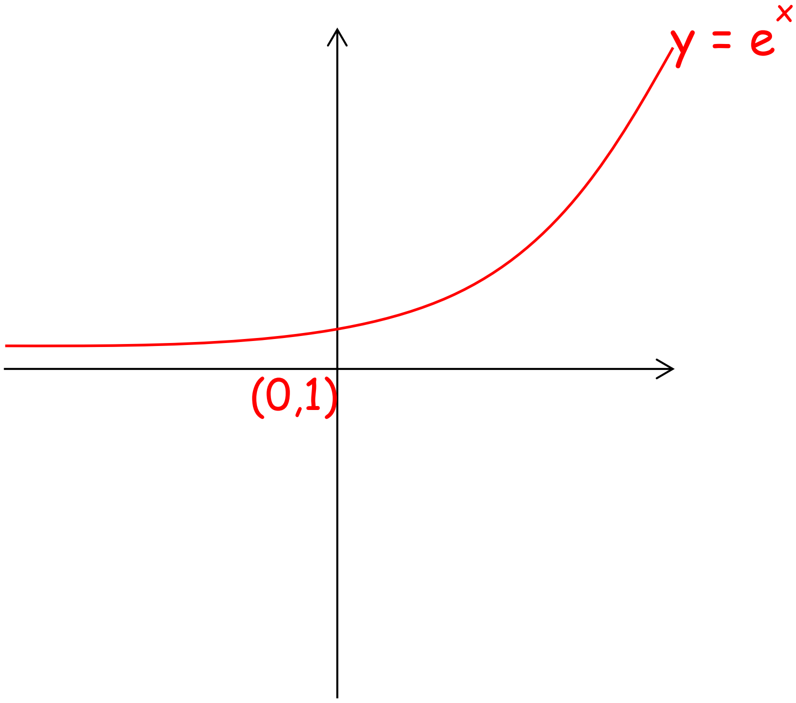
1) Differentiate the following with respect to x:

a) $y = e^{2x}$

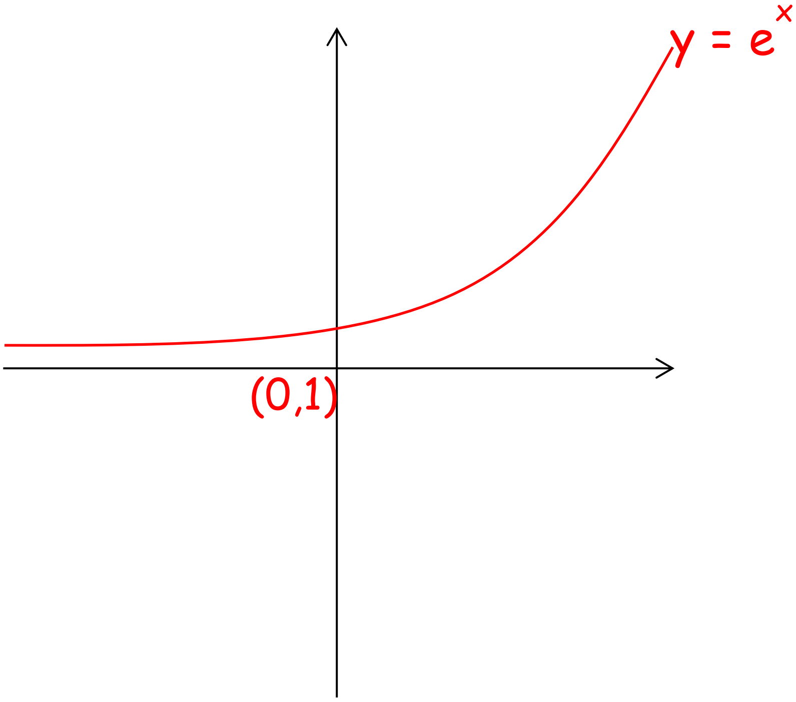
b) $y = e^{-\frac{1}{2}x}$

c) $y = 3e^{2x}$

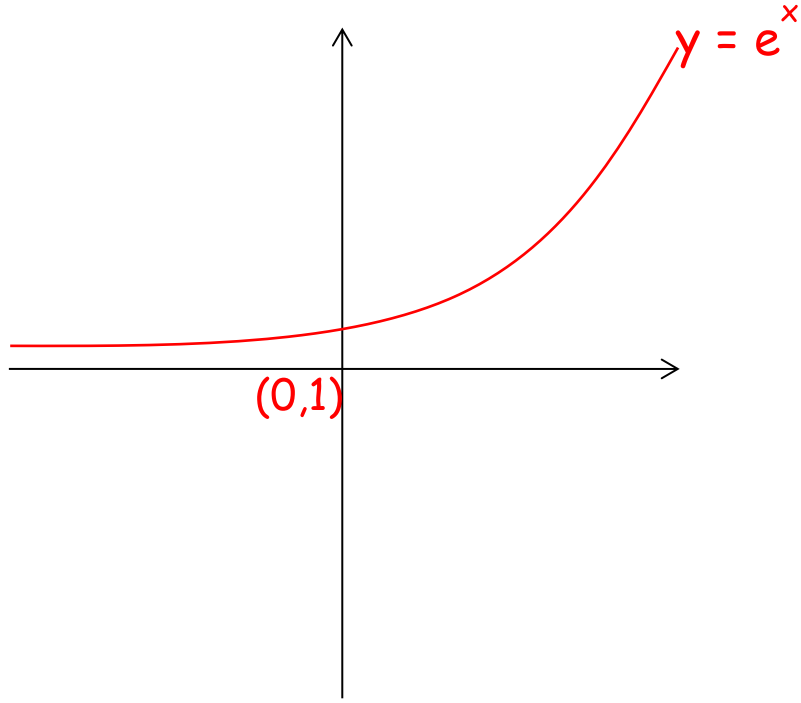
2. a) Sketch $y = 2e^x$



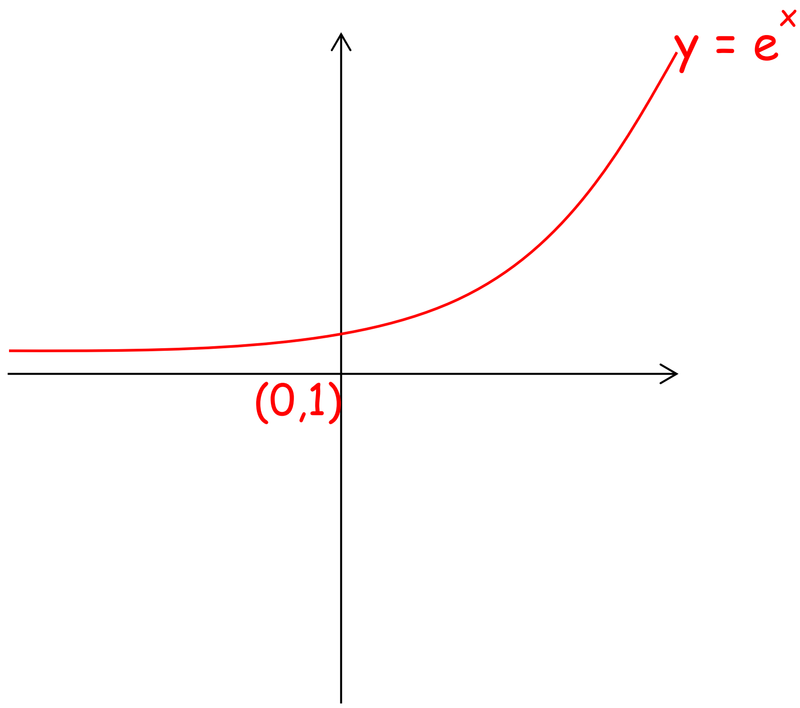
b) Sketch $y = e^x + 2$



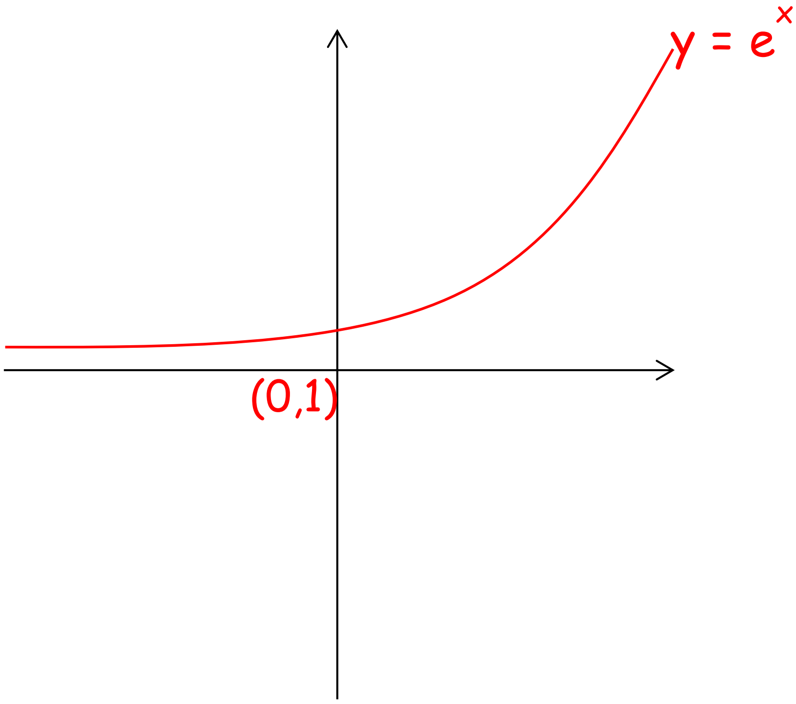
c) Sketch $y = -e^x$



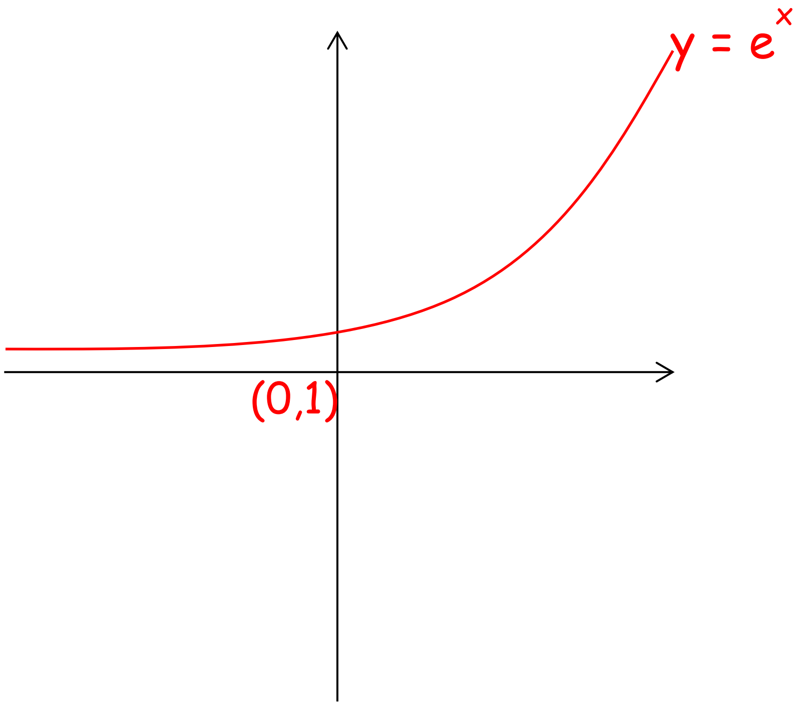
d) Sketch $y = e^{2x}$



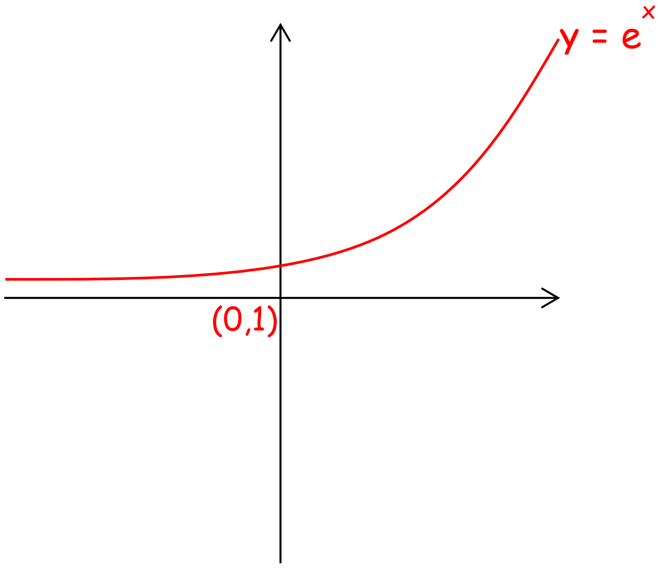
e) Sketch $y = e^{x+1}$

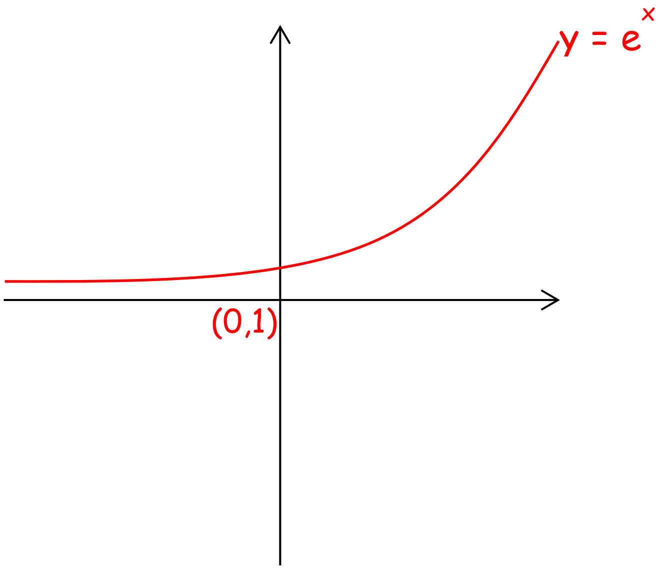


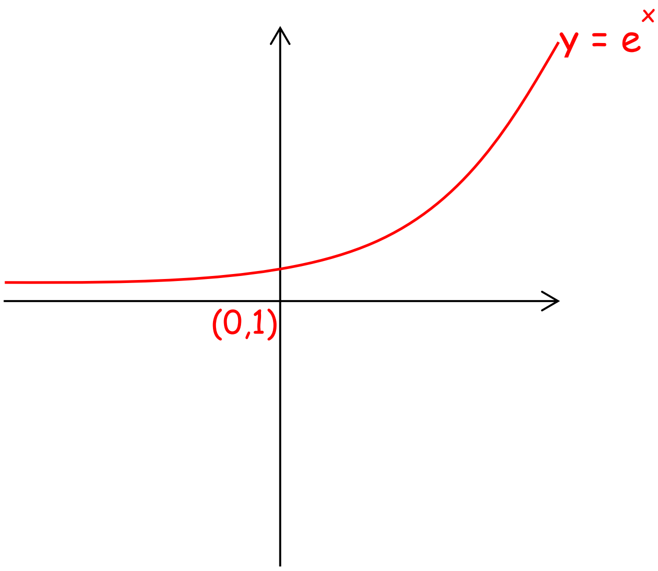
f) Sketch $y = 10e^{-x}$



g) Sketch $y = 3 + 4e^{0.5x}$







14C Modelling with e

1. The density of a pesticide in a section of field, $P\ mg/m^2$, can be modelled by the equation:
$$P = 160e^{-0.006t}$$
In this case, t is the time in days since the pesticide was first applied.

a) Estimate the density of the pesticide after 15 days

b) Interpret the meaning of the 160 in this model

c) Find $\frac{dP}{dt}$

d) Interpret the significance of the sign of your answer to part c

e) Sketch the graph of P against t.

14D Introducing Logorithms

1. Write $2^5 = 32$ as a logarithm

2. Write as a logarithm:

a) $10^3 = 1000$

b) $5^4 = 625$

c) $2^{10} = 1024$

3. Find the value of:

a) $\log_3(81)$

b) $\log_4(0.25)$

c) $\log_{0.5}(4)$

d) $\log_a(a^5)$

<u>14E Laws of Logs</u>

1. Write each of these as a single logarithm:

a) $\log_3(6) + \log_3(7)$

b) $\log_2(15) - \log_2(3)$

c) $2\log_5(3) + 3\log_5(2)$

d) $\log_{10}(3) - \log_{10}(\frac{1}{2})$

2. Write in terms of $\log_a x$, $\log_a y$ and $\log_a z$

a) $\log_a(x^2yz^3)$

b) $\log_a(\frac{x}{y^3})$

c) $\log_a (\frac{x\sqrt{y}}{z})$

d) $\log_a (\frac{x}{a^4})$

3. Solve the equation:

$$2log_2x = 8$$

4. Solve the equation:

$$log_{10}4 + 2log_{10}x = 2$$

5. Solve the equation:

$$log_3(x + 11) - log_3(x - 5) = 2$$

14F Solving Equations with Logs

1. $3^x = 20$

2. $7^{x+1} = 3^{x+2}$

3. $5^{2x} + 7\,(5^{x}) - 30 = 0$

14G ln, the 'Natural log'

1. Solve the equation $e^x = 5$

2. Solve the equation $lnx = 3$

3. Solve the equation $e^{2x+3} = 7$

4. Solve the equation $2lnx + 1 = 5$

5. Solve the equation $e^{2x} + 5e^{x} = 14$

14H Exponentials in Data

$$y = ax^n$$

$$y = ab^x$$

1. The data shows the rank (by size) and population of some UK cities.

   The relationship between P and R can be modelled by the formula:

$$P = aR^n$$

   Where a and n are constants.

a) Draw a table giving values of $logR$ and $logP$ to 2 decimal places

City	Birmingham	Leeds	Glasgow	Sheffield	Bradford
Rank, R	2	3	4	5	6
Population, P	1,000,000	730,000	620,000	530,000	480,000

City	Birmingham	Leeds	Glasgow	Sheffield	Bradford

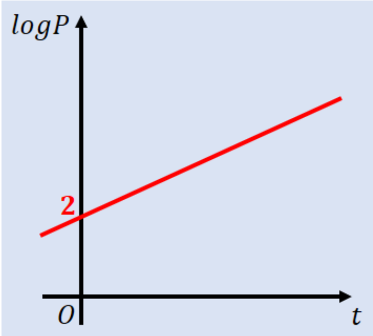
b) Plot a graph of $logR$ against $logP$ using the values from your table, and draw a line of best fit

c) Use your graph to estimate the values of a and n to two significant figures

2. The graph shown represents the growth of a population of bacteria, P over a period of t hours. The graph has a gradient of 0.6 and meets the vertical axis at (0,2) as shown.

A scientist suggests that this growth can be modelled by the equation $P = ab^t$, where a and b are constants to be found.

a) Write down an equation for the line



b) Using your answer to part a or otherwise, find the values of a and b, giving them to 3sf where necessary

c) Interpret the meaning of the constant a in this model