Maxima and Minima Problems

Recall from Pure that at minimum/maximum points, the gradient is 0. We could therefore for example find where the velocity is minimum/maximum by finding when $\frac{dv}{dt} = 0$ (i.e. when the acceleration is 0). Similarly, we can find the maximum and minimum values for displacement and acceleration.

Example

A particle P, moves in a straight line such that its velocity, $v\ ms^{-1}$ at time $t\ s$, is given by:

$$v = 5 - 9t + 6t^2 - t^3 \qquad \text{where } 0 \leq t \leq 4$$

a) Find the difference between the maximum and minimum velocities over this time interval
b) Sketch a velocity-time graph for the motion of P
c) Find the maximum acceleration over this time interval

Test Your Understanding

A dolphin escapes from Seaworld and its velocity as it speeds away from the park, is $t^3 - 9t^2 - 48t + 500$ (in ms^{-1}), until it reaches its maximum velocity, and then subsequently remains at this velocity.

(a) When does the dolphin reach its maximum velocity?

(b) What is this maximum velocity?

Test Your Understanding *(EdExcel M2 June 2013 Q3a and b)*

A particle P moves on the x-axis. At time t seconds the velocity of P is $v\,\text{m s}^{-1}$ in the direction of x increasing, where

$$v = 2t^2 - 14t + 20, \qquad t \geqslant 0$$

Find

(a) the times when P is instantaneously at rest,

(3)

(b) the greatest speed of P in the interval $0 \leqslant t \leqslant 4$

(5)

Test Your Understanding

A particle P, moves in a straight line. After t seconds, its distance, s m from its starting point A, when t = 0, is given by:

$$s = 2t^3 - 9t^2 + 12t \qquad \text{where } t \geq 0$$

a) Show that the particle never returns to its starting point

b) Find the distances from A at which the particle is instantaneously at rest

c) Find the acceleration of the particle at time t = 3s

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