

# Chapter 11 - Mechanics

## Variable Acceleration

### Chapter Overview

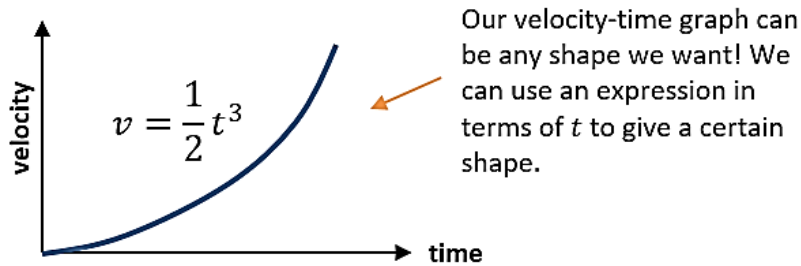
1. Functions of Time
2. Using Differentiation
3. Maxima and Minima Problems
4. Using Integration
5. Constant Acceleration Formulae

| Topics  | What students need to learn: |  |
|---|------------------------------|--|
|   | Content                      | Guidance   |
| <b>7</b><br><b>Kinematics</b><br><i>continued</i> |                              |  |
|   | 7.4                          | Use calculus in kinematics for motion in a straight line:<br>$v = \frac{dr}{dt}, \quad a = \frac{dv}{dt} = \frac{d^2r}{dt^2}$ $r = \int v \, dt, \quad v = \int a \, dt$ |



## 1. Functions of Time

Up to now, the acceleration has always been constant in any particular period of time. However, it's possible to specify either the displacement, velocity or acceleration as any function of time (i.e. an expression in terms of  $t$ ). This allows the acceleration to constantly change.



### Example

The velocity-time graph of a body is shown above, where  $v = \frac{1}{2}t^3$ .

- What is the velocity after 4 seconds have elapsed?
- How many seconds have elapsed when the velocity of the body is  $108 \text{ ms}^{-1}$ ?

**Example** *(Textbook)*

A body moves in a straight line such that its velocity,  $v \text{ m s}^{-1}$ , at time  $t$  seconds is given by:

$$v = 2t^2 - 16t + 24, \text{ for } t \geq 0$$

Find:

- a the initial velocity
- b the values of  $t$  when the body is instantaneously at rest
- c the value of  $t$  when the velocity is  $64 \text{ m s}^{-1}$
- d the greatest speed of the body in the interval  $0 \leq t \leq 5$ .

**Watch out** You need to find the greatest **speed**.

This could occur when the velocity is positive or negative, so find the range of values taken by  $v$  in the interval  $0 \leq t \leq 5$ .