Forces as Vectors

Forces have direction, and therefore we can naturally write them as vectors, either in $\boldsymbol{i}$-$\boldsymbol{j}$ notation or as column vectors.

> Add the vectors of two or more forces to find the resultant force.

Example

The forces ($3\boldsymbol{i} - 4\boldsymbol{j}$), ($2\boldsymbol{i} + 5\boldsymbol{j}$) and ($a\boldsymbol{i} + b\boldsymbol{j}$) act on a particle in equilibrium. Find the values of a and b.

If the particle is in equilibrium, what is the value of the resultant force?

We can use Pythagoras and trignometry to find the magnitude and bearing of a force when it is in vector form.

Example

The vector i is due east and j due north. A particle begins at rest at the origin. It is acted on by three forces $(2\boldsymbol{i} + \boldsymbol{j})$ N, $(3\boldsymbol{i} - 2\boldsymbol{j})$ N and $(-\boldsymbol{i} + 4\boldsymbol{j})$ N.

(a) Find the resultant force in the form $p\boldsymbol{i} + q\boldsymbol{j}$.

(b) Work out the magnitude and bearing of the resultant force.

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Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ acting on a particle P are given by

$$\mathbf{F}_1 = (7\mathbf{i} - 9\mathbf{j}) \text{ N}$$
$$\mathbf{F}_2 = (5\mathbf{i} + 6\mathbf{j}) \text{ N}$$
$$\mathbf{F}_3 = (p\mathbf{i} + q\mathbf{j}) \text{ N}$$

where p and q are constants.

Given that P is in equilibrium,

(a) find the value of p and the value of q.

(3)

The force $\mathbf{F}_3$ is now removed. The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$.
Find

(b) the magnitude of $\mathbf{R}$,

(2)

(c) the angle, to the nearest degree, that the direction of $\mathbf{R}$ makes with $\mathbf{j}$.

(3)

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A particle is acted upon by two forces $\mathbf{F}_1$ and $\mathbf{F}_2$, given by

$\mathbf{F}_1 = (\mathbf{i} - 3\mathbf{j})$ N,

$\mathbf{F}_2 = (p\mathbf{i} + 2p\mathbf{j})$ N, where p is a positive constant.

(a) Find the angle between $\mathbf{F}_2$ and $\mathbf{j}$.

(2)

The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$. Given that $\mathbf{R}$ is parallel to $\mathbf{i}$,

(b) find the value of p.

(4)

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