A small stone of mass 0.5 kg is thrown vertically upwards from a point A with an initial speed of $25\,\text{m}\,\text{s}^{-1}$. The stone first comes to instantaneous rest at the point B which is 20 m vertically above the point A. As the stone moves it is subject to air resistance. The stone is modelled as a particle.

(a) Find the energy lost due to air resistance by the stone, as it moves from A to B.

(3)

The air resistance is modelled as a constant force of magnitude R newtons.

(b) Find the value of R.

(2)

(c) State how the model for air resistance could be refined to make it more realistic.

(1)

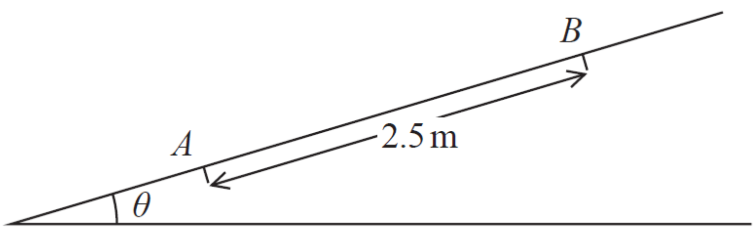


Figure 1

Figure 1 shows a ramp inclined at an angle θ to the horizontal, where $\sin\theta = \frac{2}{7}$

A parcel of mass 4 kg is projected, with speed $5\,\text{m}\,\text{s}^{-1}$, from a point A on the ramp.
The parcel moves up a line of greatest slope of the ramp and first comes to instantaneous rest at the point B, where $AB = 2.5$ m.
The parcel is modelled as a particle.

The total resistance to the motion of the parcel from non-gravitational forces is modelled as a constant force of magnitude R newtons.

(a) Use the work-energy principle to show that $R = 8.8$

(4)

After coming to instantaneous rest at B, the parcel slides back down the ramp. The total resistance to the motion of the particle is modelled as a constant force of magnitude 8.8 N.

(b) Find the speed of the parcel at the instant it returns to A.

(3)

(c) Suggest two improvements that could be made to the model.

(2)

A parcel of mass 5 kg is projected with speed $8\,\text{m}\,\text{s}^{-1}$ up a line of greatest slope of a fixed rough inclined ramp.

The ramp is inclined at angle α to the horizontal, where $\sin\alpha = \frac{1}{7}$

The parcel is projected from the point A on the ramp and comes to instantaneous rest at the point B on the ramp, where $AB = 14\,\text{m}$.

The coefficient of friction between the parcel and the ramp is μ.

In a model of the parcel's motion, the parcel is treated as a particle.

(a) Use the work-energy principle to find the value of μ.

(5)

(b) Suggest one way in which the model could be refined to make it more realistic.

(1)

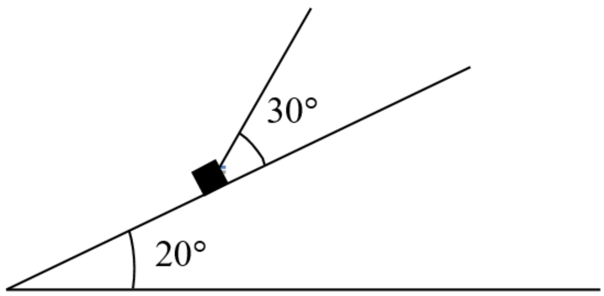


Figure 1

A small box of mass 3 kg moves on a rough plane which is inclined at an angle of 20° to the horizontal. The box is pulled up a line of greatest slope of the plane using a rope which is attached to the box. The rope makes an angle of 30° with the plane, as shown in Figure 1. The rope lies in the vertical plane which contains a line of greatest slope of the plane. The coefficient of friction between the box and the plane is 0.3. The tension in the rope is 25 N.

The box is modelled as a particle, the rope is modelled as a light inextensible string and air resistance is ignored.

Using the model,

(a) find the acceleration of the box.

(7)

(b) Suggest one improvement to the model that would make it more realistic.

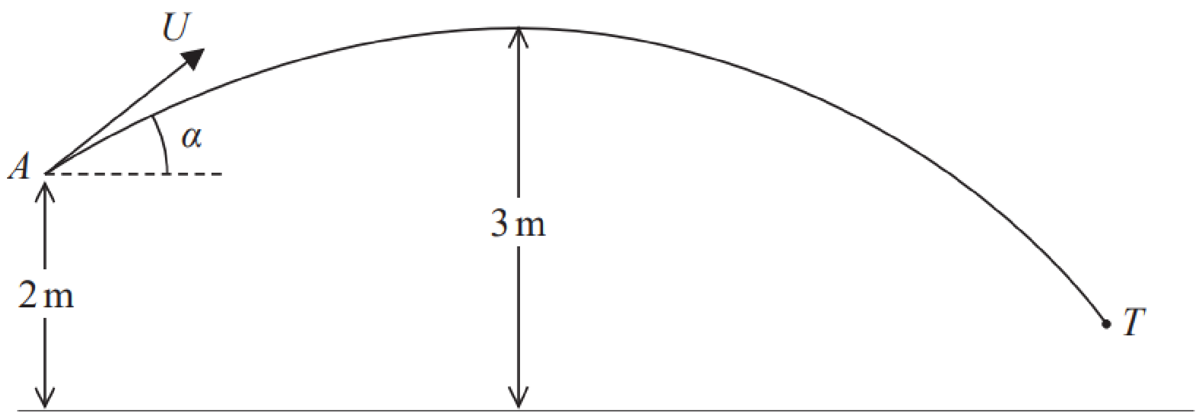
(1)

The rope now breaks and the box slows down and comes to rest.

(c) Show that, after the box comes to rest, it immediately starts to move down the plane.

(3)

(Total 11 marks)



A boy throws a ball at a target. At the instant when the ball leaves the boy's hand at the point A, the ball is 2 m above horizontal ground and is moving with speed U at an angle α above the horizontal.

In the subsequent motion, the highest point reached by the ball is 3 m above the ground.
The target is modelled as being the point T, as shown in Figure 4.
The ball is modelled as a particle moving freely under gravity.

Using the model,

(a) show that $U^2 = \dfrac{2g}{\sin^2 \alpha}$.

(2)

The point T is at a horizontal distance of 20 m from A and is at a height of 0.75 m above the ground. The ball reaches T without hitting the ground.

(b) Find the size of the angle α

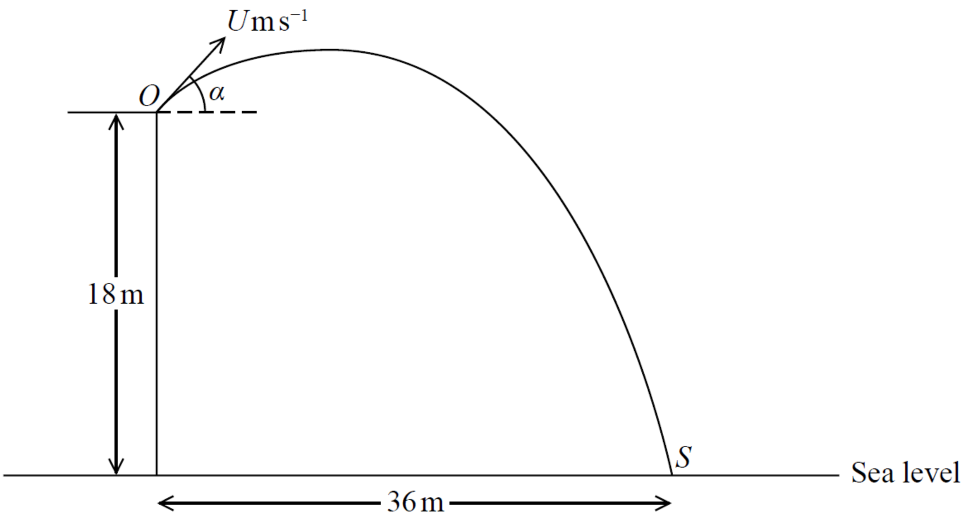
(9)

(c) State one limitation of the model that could affect your answer to part (b).

(1)

(d) Find the time taken for the ball to travel from A to T.

(3)



A boy throws a stone with speed U m s^{-1} from a point O at the top of a vertical cliff.
The point O is 18 m above sea level.

The stone is thrown at an angle α above the horizontal, where $\tan \alpha = \frac{3}{4}$.

The stone hits the sea at the point S which is at a horizontal distance of 36 m from the foot of the cliff, as shown in Figure 2.
The stone is modelled as a particle moving freely under gravity with $g = 10$ m s^{-2}

Find

(a) the value of U,

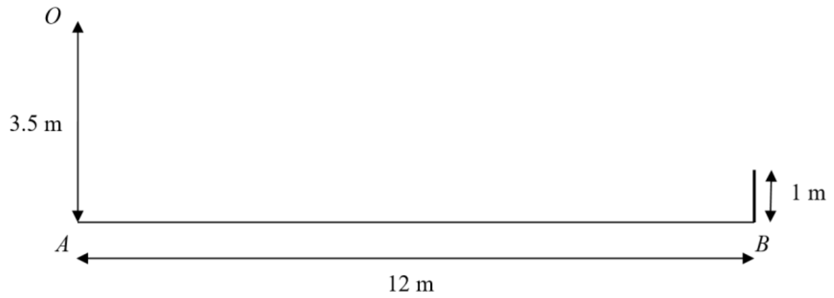
(6)

(b) the speed of the stone when it is 10.8 m above sea level, giving your answer to 2 significant figures.

(5)

(c) Suggest two improvements that could be made to the model.

(2)



A tennis player serves a ball so as to pass over the net. The ball is given an initial velocity of 45 m s^{-1} in a direction 10° below the horizontal. The ball is struck at a point O which is 3.5 m vertically above the point A which is on horizontal ground. The bottom of the net is the point B which is on the ground and AB = 12 m. The height of the net is 1 m, as shown in Figure 3.

The ball is modelled as a particle moving freely under gravity. The ball passes over the net at a point which is vertically above B.

Using the model, find

(a) in centimetres to 2 significant figures, the distance between the ball and the top of the net, as the ball passes over the net,

(8)

(b) to 2 significant figures, the speed of the ball as it passes over the net.

(4)

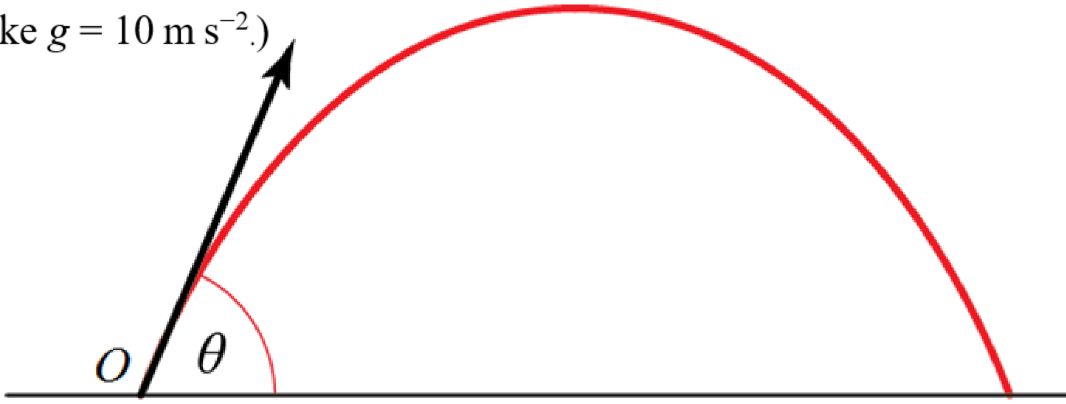
(c) State two limitations of the model that could affect the reliability of your answers.

(2)

(Total 14 marks)

A ball is launched from the origin with speed 1 m s^{-1}. Its velocity vector makes an angle θ above the horizontal. It travels over flat ground and is modelled as a particle moving freely under gravity, as shown in Figure 4.

(In this question, take $g = 10 \text{ m s}^{-2}$.)



(a) Find the horizontal and vertical displacements of the particle at time t seconds. You should give your answer in terms of θ and t.

(4)

(b) Show that the horizontal distance travelled by the particle before it hits the ground is $\dfrac{\sin 2\theta}{10}$

(5)

(c) Find the value θ for which the horizontal distance travelled is a maximum.

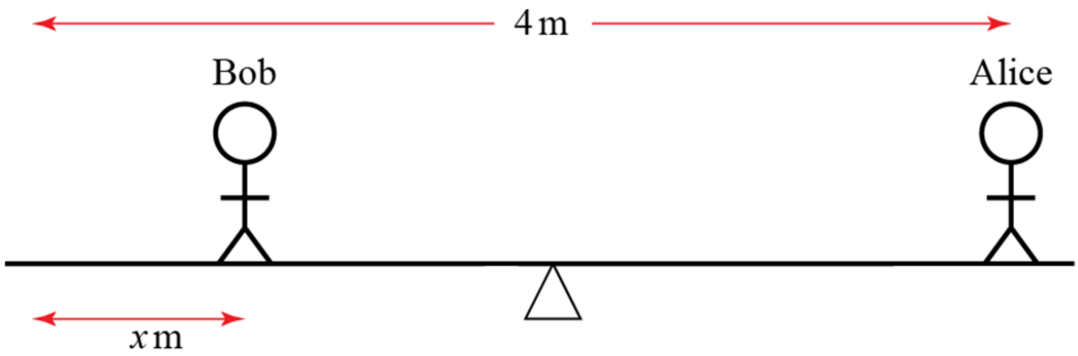
(2)

(d) Describe one limitation of this model.

(1)

(Total 12 marks)

Figure 3 shows Alice, who weighs 50 kg, sitting on the right-hand end of a light see-saw. Bob, who weighs 80 kg, stands on the opposite side at a distance x m from the end. The length of the see-saw is 4 m and it pivots about its centre.



(a) Draw a diagram showing the forces acting on the see-saw due to the two people. Label the value of each force in newtons.

(2)

(b) Write down the total clockwise moment about the centre in terms of x.

(5)

(c) Find the value of x for which the see-saw is in equilibrium.

(2)

(d) Given that Bob remains on the opposite side to Alice, describe with inequalities the range of x for which the see-saw tilts towards Alice.

(2)

(e) Describe one limitation of this model.

(1)

(Total 12 marks)