QQQ – PureYr2 - Chapter 9 – Differentiation (v2)

Total Marks: 52

(52 = Platinum, 47 = Gold, 42 = Silver, 36 = Bronze)

1.

The current, I amps, in an electric circuit at time t seconds is given by

$$I = 16 - 16(0.5)^t, \qquad t \geq 0.$$

Use differentiation to find the value of $\frac{\mathrm{d}I}{\mathrm{d}t}$ when $t = 3$.

Give your answer in the form ln a, where a is a constant.

(5)

2.

(*a*) Differentiate with respect to x

(i) $x^2 e^{3x+2}$,

(4)

(ii) $\frac{\cos(2x^3)}{3x}$.

(4)

(*b*) Given that $x = 4 \sin(2y + 6)$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x.

(5)

3.

Given that $x = \sec 4y$, find

(a) $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of y.

(2 marks)

(b) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k}{x\sqrt{x^2 - 1}}$, where k is a constant which should be found.

(3 marks)

4.

The curve C has equation $y = x^3 + 6x^2 - 12x + 6$.

(a) Show that C is concave on the interval [–5, –3].

(3 marks)

(b) Find the coordinates of the point of inflection.

(3 marks)

5.

A curve C has equation $4^x = 2xy$ for $x > 0$

Find the exact value of $\frac{dy}{dx}$ at the point C with coordinates (2, 4).

(5 marks)

6.

A curve has parametric equations $x = \cos 2t$, $y = \sin t, -\pi \le t \le \pi$.

(a) Find an expression for $\frac{dy}{dx}$ in terms of t.
Leave your answer as a single trigonometric ratio.

(3 marks)

(b) Find an equation of the normal to the curve at the point A where $t = -\frac{5\pi}{6}$.

(5 marks)

7.

The volume of a sphere V cm^3 is related to its radius r cm by the formula $V = \frac{4}{3}\pi r^3$. The surface area of the sphere is also related to the radius by the formula $S = 4\pi r^2$. Given that the rate of decrease in surface area, in cm^2 s^{-1}, is $\frac{dS}{dt} = -12$,

find the rate of decrease of volume $\frac{dV}{dt}$

(4 marks)

8.

(a) Given that f(x) = sin x, show that

$$f'(x) = \lim_{h \to 0}\left(\left(\frac{\cos h - 1}{h}\right)\sin x + \frac{\sin h}{h}\cos x\right)$$

(4 marks)

(b) Hence prove that f′(x) = cos x.

(2 marks)

Solutions (all questions © Edexcel)

q1

	$\frac{dI}{dt} = -16\ln(0.5)0.5^t$	M1 A1
At $t = 3$	$\frac{dI}{dt} = -16\ln(0.5)0.5^3$	M1
	$= -2\ln 0.5 = \ln 4$	M1 A1
		[5]

q2

(a) (i)	$\frac{d}{dx}\left(e^{3x+2}\right) = 3e^{3x+2} \quad \left(\text{or } 3e^2e^{3x}\right)$	At any stage	B1
	$\frac{dy}{dx} = 3x^2 e^{3x+2} + 2x e^{3x+2}$	Or equivalent	M1 A1+A1
			(4)
(ii)	$\frac{d}{dx}\left(\cos\left(2x^3\right)\right) = -6x^2\sin\left(2x^3\right)$	At any stage	M1 A1
	$\frac{dy}{dx} = \frac{-18x^3\sin\left(2x^3\right) - 3\cos\left(2x^3\right)}{9x^2}$		M1 A1 **(4)**
Alternatively using the product rule for second M1 A1	$y = (3x)^{-1}\cos\left(2x^3\right)$		
	$\frac{dy}{dx} = -3(3x)^{-2}\cos\left(2x^3\right) - 6x^2(3x)^{-1}\sin\left(2x^3\right)$		
		Accept equivalent unsimplified forms	

(b)	$1 = 8\cos(2y+6)\frac{dy}{dx} \quad \text{or} \quad \frac{dx}{dy} = 8\cos(2y+6)$	M1
	$\frac{dy}{dx} = \frac{1}{8\cos(2y+6)}$	M1 A1
	$\frac{dy}{dx} = \frac{1}{8\cos\left(\arcsin\left(\frac{x}{4}\right)\right)} \quad \left(= (\pm)\frac{1}{2\sqrt{(16-x^2)}}\right)$	M1 A1 **(5)**
		[13]

q3

Differentiates $x = \sec 4y$ to obtain $\frac{dx}{dy} = 4\sec 4y\tan 4y$	**M1**	1.1b	6th Differentiate reciprocal and inverse trigonometric functions.
Writes $\frac{dy}{dx} = \frac{1}{4\sec 4y\tan 4y}$	**A1**	1.1b	
	(2)		
Use the identity $\tan^2 A + 1 = \sec^2 A$ to write $\tan 4y = \sqrt{\sec^2 4y - 1} = \sqrt{x^2 - 1}$	**M1**	2.2a	6th Differentiate reciprocal and inverse trigonometric functions.
Attempts to substitute $\sec 4y = x$ and $\tan 4y = \sqrt{x^2 - 1}$ into $\frac{dy}{dx} = \frac{1}{4\sec 4y\tan 4y}$	**M1**	2.2a	
Correctly substitutes to find $\frac{dy}{dx} = \frac{1}{4x\sqrt{x^2-1}}$ and states $k = \frac{1}{4}$	**A1**	1.1b	
	(3)		

(5 marks)

q4

Finds $\frac{dy}{dx} = 3x^2 + 12x - 12$	**M1**	1.1b	7th Use second derivatives to solve problems of concavity, convexity and points of inflection.
Finds $\frac{d^2y}{dx^2} = 6x + 12$	**M1**	1.1b	
States that $\frac{d^2y}{dx^2} = 6x + 12 \leq 0$ for all $-5 \leqslant x \leqslant -3$ and concludes this implies C is concave over the given interval.	**B1**	3.2a	
	(3)		
States or implies that a point of inflection occurs when $\frac{d^2y}{dx^2} = 0$	**M1**	3.1a	7th Use second derivatives to solve problems of concavity, convexity and points of inflection.
Finds $x = -2$	**A1**	1.1b	
Substitutes $x = -2$ into $y = x^3 + 6x^2 - 12x + 6$, obtaining $y = 46$	**A1**	1.1b	
	(3)		

(6 marks)

q5

Differentiates 4^x to obtain $4^x \ln 4$	**M1**	1.1b	7th Differentiate simple functions defined implicitly.
Differentiates $2xy$ to obtain $2x\frac{dy}{dx} + 2y$	**M1**	2.2a	
Rearranges $4^x \ln 4 = 2x\frac{dy}{dx} + 2y$ to obtain $\frac{dy}{dx} = \frac{4^x \ln 4 - 2y}{2x}$	**A1**	1.1b	
Makes an attempt to substitute (2, 4)	**M1**	1.1b	
States fully correct final answer: $4\ln 4 - 2$ Accept $\ln 256 - 2$	**A1**	1.1b	

(5 marks)

q6

Finds $\frac{dx}{dt} = -2\sin 2t$ and $\frac{dy}{dt} = \cos t$	**M1**	1.1b	6th Differentiate simple functions defined parametrically including application to tangents and normals.
Writes $-2\sin 2t = -4\sin t \cos t$	**M1**	2.2a	
Calculates $\frac{dy}{dx} = \frac{\cos t}{-4\sin t \cos t} = -\frac{1}{4}\operatorname{cosec} t$	**A1**	1.1b	
	(3)		

Evaluates $\frac{dy}{dx}$ at $t=-\frac{5\pi}{6}$ $\frac{dy}{dx}=\frac{-1}{4\sin\left(-\frac{5\pi}{6}\right)}=\frac{1}{2}$	**A1 ft**	1.1b	6th Differentiate simple functions defined parametrically including application to tangents and normals.
Understands that the gradient of the tangent is $\frac{1}{2}$, and then the gradient of the normal is −2.	**M1 ft**	1.1b	
Finds the values of x and y at $t=-\frac{5\pi}{6}$ $x=\cos\left(2\times-\frac{5\pi}{6}\right)=\frac{1}{2}$ and $y=\sin\left(-\frac{5\pi}{6}\right)=-\frac{1}{2}$	**M1 ft**	1.1b	
Attempts to substitute values into $y-y_1=m(x-x_1)$ For example, $y+\frac{1}{2}=-2\left(x-\frac{1}{2}\right)$ is seen.	**M1 ft**	2.2a	
Shows logical progression to simplify algebra, arriving at: $y=-2x+\frac{1}{2}$ or $4x+2y-1=0$	**A1**	2.4	
	(5)		

(8 marks)

Q7

Recognises the need to use the chain rule to find $\frac{dV}{dt}$ For example $\frac{dV}{dt}=\frac{dV}{dr}\times\frac{dr}{dS}\times\frac{dS}{dt}$ is seen.	**M1**	3.1a	8th Construct differential equations in a range of contexts.
Finds $\frac{dV}{dr}=4\pi r^2$ and $\frac{dS}{dr}=8\pi r$	**M1**	2.2a	
Makes an attempt to substitute known values. For example, $\frac{dV}{dt}=\frac{4\pi r^2}{1}\times\frac{1}{8\pi r}\times\frac{-12}{1}$	**M1**	1.1b	
Simplifies and states $\frac{dV}{dt}=-6r$	**A1**	1.1b	

(4 marks)

Question 8

States $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{x+h-x}$	**M1**	3.1b	5th Differentiate simple trigonometric functions.
Makes correct substitutions: $f'(x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$	**M1**	1.1b	
Uses the appropriate trigonometric addition formula to write $f'(x) = \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$	**M1**	2.2a	
Groups the terms appropriately $f'(x) = \lim_{h \to 0}\left(\left(\frac{\cos h - 1}{h}\right)\sin x + \left(\frac{\sin h}{h}\right)\cos x\right)$	**A1**	2.2a	
	(4)		
Explains that as $h \to 0$, $\frac{\cos h - 1}{h} \to 0$ and $\frac{\sin h}{h} \to 1$	**M1**	3.2b	5th Differentiate simple trigonometric functions.
Concludes that this leaves $0 \times \sin x + 1 \times \cos x$ So if $f(x) = \sin x$, $f'(x) = \cos x$	**A1**	3.2b	
	(2)		

(6 marks)