**Pure 2**

**Vectors**

Chapter Overview

1:: Distance between two points.

2:: $i,j,k$ notation for vectors

3:: Magnitude of a 3D vector and using it to find angle between vector and a coordinate axis.

4:: Solving Geometric Problems

5:: Application to Mechanics

**Distance from the origin and magnitude of a vector**

 In 2D, how did we find the distance from a point to the origin?

 



**The magnitude of a vector** $a=\left(\begin{matrix}x\\y\\z\end{matrix}\right)$**:**

$$\left|a\right|=\sqrt{x^{2}+y^{2}+z^{2}}$$

**And the distance of** $(x,y,z)$ **from the origin is** $\sqrt{x^{2}+y^{2}+z^{2}}$

**Distance between two 3D points**

 

How do we find the distance between $P$ and $Q$?

**The distance between two points is:**

$$d=\sqrt{\left(Δx\right)^{2}+\left(Δy\right)^{2}+\left(Δz\right)^{2}}$$

 $Δx$ **means “change in** $x$**”**

**Quickfire Questions:**

Distance of $(4,0,-2)$ from the origin:

$$\left|\left(\begin{matrix}5\\4\\-1\end{matrix}\right)\right|=$$

Distance between $(0,4,3)$ and $\left(5,2,3\right).$

Distance between $(1,1,1)$ and $\left(2,1,0\right).$

Distance between $(-5,2,0)$ and $\left(-2,-3,-3\right).$

**Tip**: Because we’re squaring, it doesn’t matter whether the change is negative or positive.

***Test Your Understanding So Far…***

[Textbook] **Find the distance from the origin to the point** $P(7,7,7)$**.**

[Textbook] **The coordinates of** $A$ **and** $B$ **are** $(5,3,-8)$ **and** $\left(1,k,-3\right)$ **respectively. Given that the distance from** $A$ **to** $B$ **is** $3\sqrt{10} $ **units, find the possible values of** $k$**.**

Ex 12A p.338

$i$**,** $j$ **and** $k$ **notation**

In 2D you were previously introduced to $i=\left(\begin{matrix}1\\0 \end{matrix}\right)$ and $j=\left(\begin{matrix}0\\1\end{matrix}\right)$ as unit vectors in each of the $x$ and $y$ directions.

It meant for example that $\left(\begin{matrix}8\\-2\end{matrix}\right)$ could be written as $8i-2j$since $8\left(\begin{matrix}1\\0 \end{matrix}\right)-2\left(\begin{matrix}0\\1\end{matrix}\right)=\left(\begin{matrix}8\\-2\end{matrix}\right)$

Unsurprisingly, **in 3D:**

$$i=\left(\begin{matrix}1\\0\\0\end{matrix}\right), j=\left(\begin{matrix}0\\1\\0\end{matrix}\right), k=\left(\begin{matrix}0\\0\\1\end{matrix}\right)$$

**Quickfire Questions**

1. Put in $i,j,k$ notation:

$$\left(\begin{matrix}1\\2\\3\end{matrix}\right)=$$

$$\left(\begin{matrix}3\\0\\-1\end{matrix}\right)=$$

1. Write as a column vector:

$$4j+k=$$

$$i-j=$$

1. If $A\left(1,2,3\right), B\left(4,0,-1\right)$ then

$$\vec{AB}=$$

1. If $a=\left(\begin{matrix}2\\3\\4\end{matrix}\right)$ and $b=\left(\begin{matrix}0\\-1\\3\end{matrix}\right)$ then $3a+2b=$

**Examples**

1. Find the magnitude of $a=2i-j+4k$ and hence find $\hat{a}$, the unit vector in the direction of $a$.
2. If $a=\left(\begin{matrix}2\\-3\\5\end{matrix}\right)$ and $b=\left(\begin{matrix}4\\-2\\0 \end{matrix}\right)$ is $2a-3b$ parallel to $4i-5k$?

**Angles between vectors and an axis**

How could you work out the angle between a vector and the $x$-axis?



* **The angle between** $a=\left(\begin{matrix}x\\y\\z\end{matrix}\right)$ **and the** $x$**-axis is:**

$$\cos(θ\_{x})=\frac{x}{\left|a\right|}$$

**and similarly for the** $y$ **and** $z$ **axes.**

[Textbook] **Find the angles that the vector** $a=2i-3j-k$ **makes with each of the positive coordinate axis.**

***Test Your Understanding***

[Textbook] **The points** $A$ **and** $B$ **have position vectors** $4i+2j+7k$ **and**

$3i+4j-k$ **relative to a fixed origin,** $O$**. Find** $\vec{AB}$ **and show that** $ΔOAB$ **is isosceles.**

**(a) Find the angle that the vector** $a=2i+j+k$ **makes with the** $x$**-axis.**

**(b) By similarly considering the angle that** $b=i+3j+2k$ **makes with the** $x$**-axis, determine the area of** $OAB$ **where** $\vec{OA}=a$ **and** $\vec{OB}=b$**. (Hint: draw a diagram)**

Ex 12B p.341-343

**Solving geometric problems**

For more general problems involving vectors, often **drawing a diagram** helps!

[Textbook] $A, B, C$ **and** $D$ **are the points** $\left(2,-5,-8\right)$**,** $\left(1,-7,-3\right), (0,15,-10)$ **and** $\left(2,19,-20\right)$ **respectively.**

1. **Find** $\vec{AB}$ **and** $\vec{DC}$**, giving your answers in the form**$pi+qj+rk$**.**
2. **Show that the lines** $AB$ **and** $DC$ **are parallel and that** $\vec{DC}=2\vec{AB}$**.**
3. **Hence describe the quadrilateral** $ABCD$**.**

[Textbook] $P, Q$ **and** $R$ **are the points** $\left(4,-9,-3\right),(7,-7,-7)$ **and** $\left(8,-2,0\right)$ **respectively. Find the coordinates of the point** $S$ **so that** $PQRS$ **forms a parallelogram.**

There are many contexts in maths where we can ‘compare coefficients’, e.g.

 $3x^{2}+5x≡A\left(x^{2}+1\right)+Bx+C$

 Comparing $x^{2}$ terms: $3=A$

We can do the same with vectors:

[Textbook] **Given that**$3i+\left(p+2\right)j+120k=pi-qj+4pqrk$**, find the values of** $p, q$ **and** $r$**.**

[Textbook] **The diagram shows a cuboid whose vertices are** $O,A,B,C,D,E,F$ **and** $G$**. Vectors** $a, b$ **and** $c$ **are the position vectors of the vertices** $A, B$ **and** $C$ **respectively. Prove that the diagonals** $OE$ **and** $BG$ **bisect each other.**



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The strategy behind this type of question is to find the point of intersection in 2 ways, and compare coefficients.

Ex 12C p.346-347

**Application to Mechanics**

Out of displacement, speed, acceleration, force, mass and time, all but mass and time are vectors. Clearly these can act in 3D space.



**Example**

[Textbook] **A particle of mass 0.5 kg is acted on by three forces.**

$$F\_{1}=\left(2i-j+2k\right) NF\_{2}=\left(-i+3j-3k\right) NF\_{3}=\left(4i-3j-2k\right) N$$

1. **Find the resultant force** $R$ **acting on the particle.**
2. **Find the acceleration of the particle, giving your answer in the form** $\left(pi+qj+rk\right)$ **ms-2.**
3. **Find the magnitude of the acceleration.**

**Given that the particle starts at rest,**

**d. Find the distance travelled by the particle in the first 6 seconds of its motion.**

Ex 12D p.348-349