A Level Mathematics

Chapter 8 - Mechanics

Further Kinematics

Chapter Overview

1. Vectors in Kinematics

2. Vector Methods with Projectiles

3. Variable Acceleration in One Dimension

4. Differentiating Vectors

5. Integrating Vectors





1. **Vectors in Kinematics**

If a particle starts from the point with position vector ***r***0, and moves with constant velocity ***v***, its displacement from its initial position at time t is given by ***v***t and it position vector ***r*** is given by:

**Example**

At time $t = 0$, where $t$ is the time (in seconds), a particle is at the point with position vector $(4i - j) $m and travels with velocity $(-2i + 2j) $ms-1. Find:

a) The position vector of the particle after $t$ seconds

b) The distance the particle is from the origin, O, after 3 seconds.

**Example**

A particle starts at a point 8m from O at an angle of 45O anti-clockwise from east and travels with a velocity $(-2i - 3j)$ ms-1, where $i$ and $j$ are unit vectors due east and north respectively.

Find the position vector of the particle after t seconds in the form $r = r\_{0} + tv$.

**Example – Using SUVAT with Vectors**

A particle is initially travelling with velocity $(-2i - 9j)$ ms-1 and 2 seconds later it has a velocity of$ (6i - 11j)$ ms-1, where $i$ and $j$ are unit vectors in the directions of the positive x- and y- axes respectively. Given that the acceleration of the particle is constant, find:

a) The acceleration

b) The magnitude of the acceleration

c) The angle that the acceleration makes with the vector $j$

**Example** *(Textbook p161 Example 3)*

An ice skater is skating on a large flat ice rink. At time $t=0$ the skater is at a fixed point $O$ and is travelling with velocity $\left(2.4i-0.6j\right)$ ms-1.

At time $t=20$ s the skater is travelling with velocity $\left(-5.6i+3.4j\right)$ ms-1.

Relative to $O$, the skater has position vector $s$ at time $t$ seconds.

Modelling the ice skater as a particle with constant acceleration, find:

1. The acceleration of the ice skater
2. An expression for $s$ in terms of $t$
3. The time at which the skater is directly north-east of $O$.

A second skater travels so that she has position vector$r=\left(1.1t-6\right)j$ m relative to $O$ at time $t$.

1. Show that the two skaters will meet.

**Test Your Understanding** *(EdExcel M1 May 2013(R) Q6)*



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1. **Vector Methods with Projectiles**

Previously we considered the initial speed of the projectile and the angle of projection. But we could also **use a velocity vector to represent the initial projection** (vectors have both direction and magnitude) and subsequent motion.

**Example**

A ball is projected from the origin with velocity $(12i + 24j)$ms-1 where $i$ and $j$ are horizontal and vertical unit vectors respectively. The particle moves freely under gravity. Find:

a) The position vector of the ball after 3s

b) The speed of the ball after 3s

c) The ball strikes the ground at point B. Determine the distance OB

**Example**

A particle *P* is projected with velocity $(4pi + 5pj) $ms-1 from a point *O* on a horizontal plane, where $i$and$ j$ are horizontal and vertical unit vectors respectively.

The particle *P* strikes the plane at the point *A,* which is 800 m from *O*.

a) Show that $p = 14$.

b) Find the time of flight from *O* to *A*.

The particle *P* passes through a point *B* with speed 60 m s-1.

c) Find the height of *B* above the horizontal plane.

**Test Your Understanding** *(EdExcel M2 Jan 2012 Q7)*



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1. **Variable Acceleration in One Dimension**

Displacement ($s$)

Velocity ($v$)

Acceleration ($a$)

Differentiate

Integrate

Don’t forget the constant of integration for indefinite integrals

**Example**

A particle is moving in a straight line with acceleration at time $t$ seconds given by

$$a=\cos(2πt ms^{-2},    t\geq 0)$$

The velocity of the particle at time $t=0$ is $\frac{1}{2π}$ ms-1. Find:

1. an expression for the velocity at time $t$ seconds
2. the maximum speed
3. the distance travelled in the first 3 seconds.

**Test Your Understanding** *(Textbook p168 Example 6)*

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A particle of mass 6kg is moving on the positive $x$-axis. At time $t$ seconds the displacement, $s$, of the particle from the origin is given by

$$s=2t^{\frac{3}{2}}+\frac{e^{-2t}}{3} m,    t\geq 0$$

1. Find the velocity of the particle when $t=1.5$.
2. Given that the particle is acted on by a single force of variable magnitude $F$ N which acts in the direction of the positive $x$-axis,
3. Find the value of $F$ when $t=2$
4. **Differentiating Vectors**

We use calculus with 2-d (and 3-d) vectors by differentiating and integrating each function of time separately:

 If $r=xi+yj$, then

**Example**

A particle $P$ of mass 0.8kg is acted on by a single force $F$ N. Relative to a fixed origin $O$, the position vector of $P$ at time $t$ seconds is $r$ metres, where

$$r=2t^{3}i+50t^{-\frac{1}{2}}j,   t\geq 0$$

Find:

1. the speed of $P$ when $t=4$
2. the acceleration of $P$ as a vector when $t=2$
3. $F$ when $t=2$.

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1. **Integrating Vectors**

We can integrate vectors by integrating each function of time separately.

Remember each component will have a constant of integration, $C=(pi + qj)$.

**Example**

A force $F$ acts on a body of mass 250g which is initially at rest at a fixed point O. If $F = ((5t - 2)i + 4tj)$N, where$ t$ is the time for which the force has been acting on the body, find expressions for:

a) The velocity vector of the body at time $t$.

b) The position vector of the body at time $t$.

**Example** *(Textbook)*

A particle $P$ is moving in a plane so that, at time $t$ seconds, its acceleration is $(4i-2tj)$ms-2. When $t=3$, the velocity of $P$ is $6i$ ms-1 and the position vector of $P$ is $(20i+3j)$ m with respect to a fixed origin $O$. Find:

1. the angle between the direction of motion of $P$ and $i$ when $t=2$
2. the distance of $P$ from $O$ when $t=0$.

**Test Your Understanding** *(EdExcel M2 Jan 2013 Q4)*



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Mixed Exercise 8 Page 177