A Level Mathematics

Chapter 8 - Mechanics

Further Kinematics

Chapter Overview

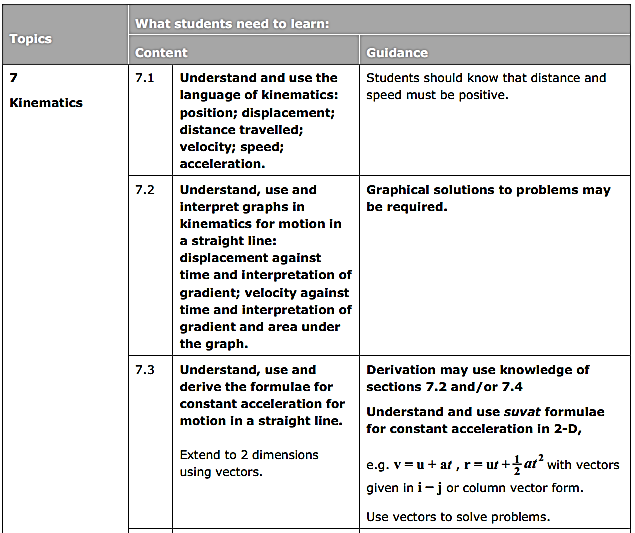
1. Vectors in Kinematics

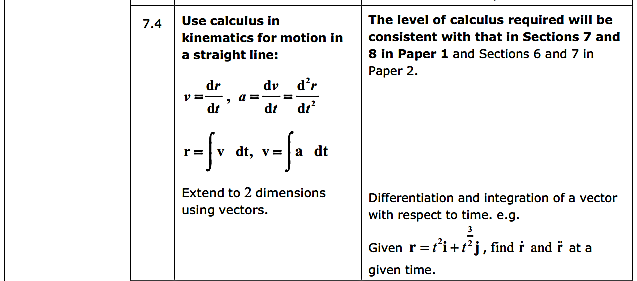
2. Vector Methods with Projectiles

3. Variable Acceleration in One Dimension

4. Differentiating Vectors

5. Integrating Vectors





1. **Vectors in Kinematics**

If a particle starts from the point with position vector ***r***0, and moves with constant velocity ***v***, its displacement from its initial position at time t is given by ***v***t and it position vector ***r*** is given by:

**Example**

At time , where is the time (in seconds), a particle is at the point with position vector m and travels with velocity ms-1. Find:

a) The position vector of the particle after seconds

b) The distance the particle is from the origin, O, after 3 seconds.

**Example**

A particle starts at a point 8m from O at an angle of 45O anti-clockwise from east and travels with a velocity ms-1, where and are unit vectors due east and north respectively.

Find the position vector of the particle after t seconds in the form .

**Example – Using SUVAT with Vectors**

A particle is initially travelling with velocity ms-1 and 2 seconds later it has a velocity of ms-1, where and are unit vectors in the directions of the positive x- and y- axes respectively. Given that the acceleration of the particle is constant, find:

a) The acceleration

b) The magnitude of the acceleration

c) The angle that the acceleration makes with the vector

**Example** *(Textbook p161 Example 3)*

An ice skater is skating on a large flat ice rink. At time the skater is at a fixed point and is travelling with velocity ms-1.

At time s the skater is travelling with velocity ms-1.

Relative to , the skater has position vector at time seconds.

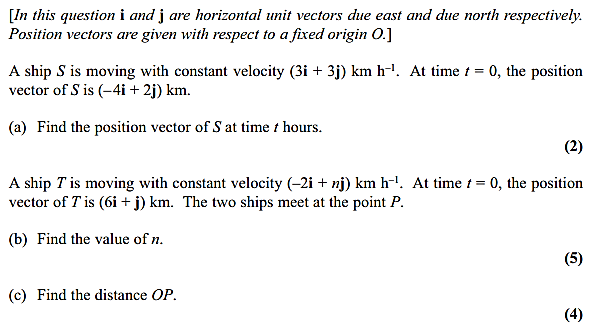
Modelling the ice skater as a particle with constant acceleration, find:

1. The acceleration of the ice skater
2. An expression for in terms of
3. The time at which the skater is directly north-east of .

A second skater travels so that she has position vector m relative to at time .

1. Show that the two skaters will meet.

**Test Your Understanding** *(EdExcel M1 May 2013(R) Q6)*



Exercise 7A Page 162

1. **Vector Methods with Projectiles**

Previously we considered the initial speed of the projectile and the angle of projection. But we could also **use a velocity vector to represent the initial projection** (vectors have both direction and magnitude) and subsequent motion.

**Example**

A ball is projected from the origin with velocity ms-1 where and are horizontal and vertical unit vectors respectively. The particle moves freely under gravity. Find:

a) The position vector of the ball after 3s

b) The speed of the ball after 3s

c) The ball strikes the ground at point B. Determine the distance OB

**Example**

A particle *P* is projected with velocity ms-1 from a point *O* on a horizontal plane, where and are horizontal and vertical unit vectors respectively.

The particle *P* strikes the plane at the point *A,* which is 800 m from *O*.

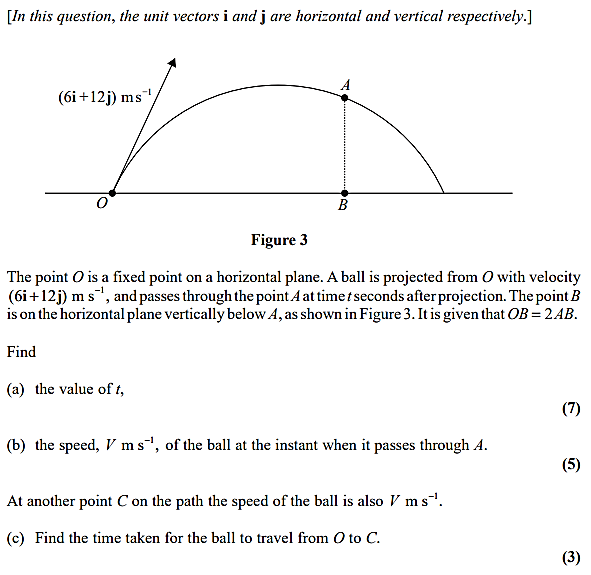
a) Show that .

b) Find the time of flight from *O* to *A*.

The particle *P* passes through a point *B* with speed 60 m s-1.

c) Find the height of *B* above the horizontal plane.

**Test Your Understanding** *(EdExcel M2 Jan 2012 Q7)*



Exercise 8B Page 166

1. **Variable Acceleration in One Dimension**

Displacement ()

Velocity ()

Acceleration ()

Differentiate

Integrate

Don’t forget the constant of integration for indefinite integrals

**Example**

A particle is moving in a straight line with acceleration at time seconds given by

The velocity of the particle at time is ms-1. Find:

1. an expression for the velocity at time seconds
2. the maximum speed
3. the distance travelled in the first 3 seconds.

**Test Your Understanding** *(Textbook p168 Example 6)*

Exercise 8C Page 168

A particle of mass 6kg is moving on the positive -axis. At time seconds the displacement, , of the particle from the origin is given by

1. Find the velocity of the particle when .
2. Given that the particle is acted on by a single force of variable magnitude N which acts in the direction of the positive -axis,
3. Find the value of when
4. **Differentiating Vectors**

We use calculus with 2-d (and 3-d) vectors by differentiating and integrating each function of time separately:

If , then

**Example**

A particle of mass 0.8kg is acted on by a single force N. Relative to a fixed origin , the position vector of at time seconds is metres, where

Find:

1. the speed of when
2. the acceleration of as a vector when
3. when .

Exercise 8D Page 171

1. **Integrating Vectors**

We can integrate vectors by integrating each function of time separately.

Remember each component will have a constant of integration, .

**Example**

A force acts on a body of mass 250g which is initially at rest at a fixed point O. If N, where is the time for which the force has been acting on the body, find expressions for:

a) The velocity vector of the body at time .

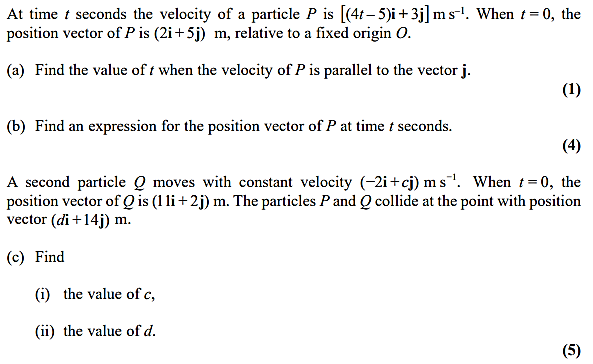
b) The position vector of the body at time .

**Example** *(Textbook)*

A particle is moving in a plane so that, at time seconds, its acceleration is ms-2. When , the velocity of is ms-1 and the position vector of is m with respect to a fixed origin . Find:

1. the angle between the direction of motion of and when
2. the distance of from when .

**Test Your Understanding** *(EdExcel M2 Jan 2013 Q4)*



Exercise 8E Page 175

Mixed Exercise 8 Page 177