Chapter 7 - Mechanics

Applications of Forces

Chapter Overview

1. Static Particles
2. Modelling with Statics
3. Friction and Static Particles
4. Static Rigid Bodies
5. Dynamics and Inclined Planes
6. Connected Particles

Topics	What students need to learn:		
	Content		Guidance
8 **Forces and Newton's laws**	8.1	**Understand the concept of a force; understand and use Newton's first law.**	**Normal reaction, tension, thrust or compression, resistance.**
	8.2	**Understand and use Newton's second law for motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors);** extend to situations where forces need to be resolved (restricted to 2 dimensions).	**Problems will involve motion in a straight line with constant acceleration in scalar form, where the forces act either parallel or perpendicular to the motion.** Extend to problems where forces need to be resolved, e.g. a particle moving on an inclined plane. **Problems may involve motion in a straight line with constant acceleration in vector form, where the forces are given in i – j form or as column vectors.**

	8.3	**Understand and use weight and motion in a straight line under gravity; gravitational acceleration, *g*, and its value in S.I. units to varying degrees of accuracy.**	**The default value of g will be 9.8 m s^{-2} but some questions may specify another value, e.g. g = 10 m s^{-2}** **The inverse square law for gravitation is not required and g may be assumed to be constant, but students should be aware that g is not a universal constant but depends on location.**
	8.4	**Understand and use Newton's third law; equilibrium of forces on a particle and motion in a straight line; application to problems involving smooth pulleys and connected particles;** resolving forces in 2 dimensions; equilibrium of a particle under coplanar forces.	**Connected particle problems could include problems with particles in contact e.g. lift problems.** Problems may be set where forces need to be resolved, e.g. at least one of the particles is moving on an inclined plane.
	8.5	Understand and use addition of forces; resultant forces; dynamics for motion in a plane.	Students may be required to resolve a vector into two components or use a vector diagram, e.g. problems involving two or more forces, given in magnitude-direction form.
8 **Forces and Newton's laws** *continued*	8.6	Understand and use the $F \leq \mu R$ model for friction; coefficient of friction; motion of a body on a rough surface; limiting friction and statics.	An understanding of $F = \mu R$ when a particle is moving. An understanding of $F \leq \mu R$ in a situation of equilibrium.
9 **Moments**	9.1	Understand and use moments in simple static contexts.	Equilibrium of rigid bodies. Problems involving parallel and non-parallel coplanar forces, e.g. ladder problems.

In this chapter, we will bring together everything that we have learned about forces: friction, resolving forces into components, Newton's 2nd law, inclined planes and connected particles, for different, common types of problems.

1. <u>Static Particles</u>

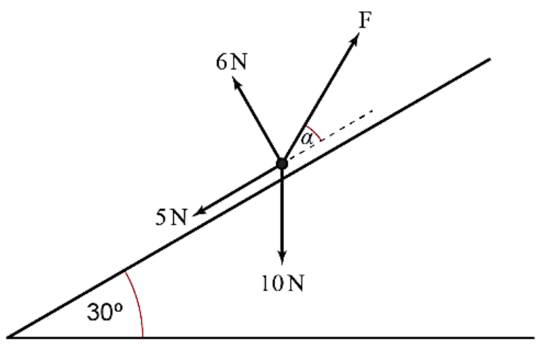
If a particle is in equilibrium, the resultant of all forces is 0 and the particle remains at rest.

- Always draw a diagram
- Resolve the forces, horizontal and vertical, or parallel and perpendicular if on an inclined plane
- In each direction, sum of components = 0
- Solve the resulting equations to find unknown forces

For particles in equilibrium, you can also use a triangle of forces.

<u>Example</u>

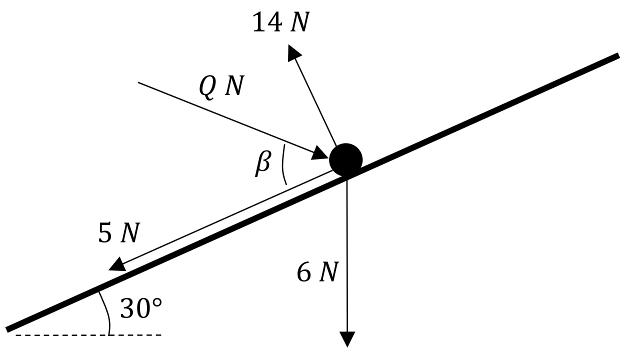
The diagram shows a particle in equilibrium under the action of four forces as shown in the diagram below. The particle rests on an inclined plane which is set at an angle of 30° to the horizontal.



Find the magnitude of force *F* and the size of the angle, α, in degrees giving both answers to two significant figures.

Test Your Understanding

The diagram shows a particle in equilibrium on an inclined plane under the forces shown.
Find the magnitude of the force Q and the size of the angle β.



Hint: Redraw the Q N force

Exercise 7A Page 130

2. Modelling with Statics

Remember to include additional forces such as weight, tension, thrust, normal reaction, friction etc.

Example

A light, inextensible string of length 50cm has its upper end fixed at a point A and comes with a particle of mass 8kg at its lower end. A horizontal force P applied to the particle keeps it in equilibrium 30cm from the vertical through A.

By resolving horizontally and vertically, find the magnitude of P and the tension in the string.

Example

A light, inextensible string passes over a smooth pulley fixed at the top of a smooth plane inclined at 30^{O} to the horizontal. A particle of mass 2kg is attached to one end of the string and hangs freely. A mass m is attached to the other end of the string and rests in equilibrium on the surface of the plane.

Calculate the normal reaction between the mass m and the plane, the tension in the string and the value of m.

Test Your Understanding *(EdExcel M1 May 2013(R) Q2)*

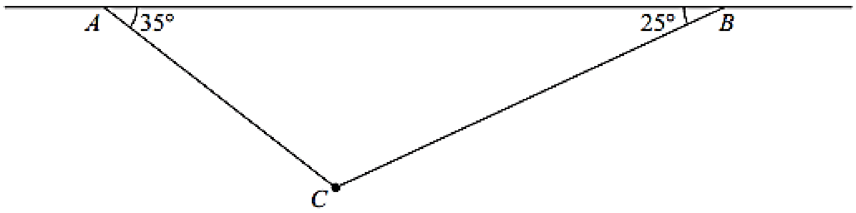


Figure 1

A particle of weight 8 N is attached at C to the ends of two light inextensible strings AC and BC. The other ends, A and B, are attached to a fixed horizontal ceiling. The particle hangs at rest in equilibrium, with the strings in a vertical plane. The string AC is inclined at 35° to the horizontal and the string BC is inclined at 25° to the horizontal, as shown in Figure 1. Find

(i) the tension in the string AC,

(ii) the tension in the string BC.

(8)

Exercise 7B Page 134

3. Friction and Static Particles

If there is no motion, the maximum frictional force, F_{max}, has not yet been reached.
When $F_{max} = \mu R$, the body is on the point of moving. This is called *limiting equilibrium*.
In Statics, the force of friction, F, is such that $\leq \mu R$, and the direction of the friction force is **opposite** to the direction in which the body would move if the friction force were absent.

Example

A 10kg truck lies on a horizontal rough floor. The coefficient of friction between the trunk and the floor is $\frac{\sqrt{3}}{4}$.

Calculate the magnitude of the force, P, which is necessary to pull the trunk horizontally if P is applied:

a) horizontally

b) at 30^{O} above the horizontal

Example – Rough Inclined Plane

A mass of 6kg rests in limiting equilibrium on a rough plane inclined at 30^{O} to the horizontal.

a) Find the coefficient of friction between the mass and the plane.

b) A horizontal force of magnitude *P* N is applied to the box. Given that the box remains in equilibrium, find the maximum possible value of *P*.

Exercise 7C Page 139

4. Static Rigid Bodies

Recall from the chapter on moments that for a stationary rigid body:

- The **resultant force is 0**.
- The **resultant moment is 0**.

The problems are the same as in the moments chapter, except now we may need to consider frictional forces.

Example

A uniform rod AB of mass 45kg and length 12m rests with the end A on rough horizontal ground. The rod rests against a smooth peg C where $AC = 8$ m. The rod is in limiting equilibrium at an angle of $15°$ to the horizontal. Find:

(a) the magnitude of the reaction of C

(b) the coefficient of friction between the rod and the ground.

Example *(EdExcel M2 Jan 2012 Q5)*

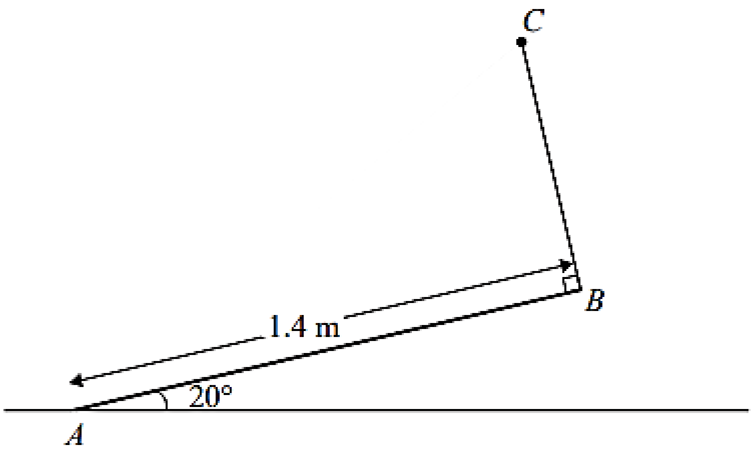


Figure 2

A uniform rod AB has mass 4 kg and length 1.4 m. The end A is resting on rough horizontal ground. A light string BC has one end attached to B and the other end attached to a fixed point C. The string is perpendicular to the rod and lies in the same vertical plane as the rod. The rod is in equilibrium, inclined at 20° to the ground, as shown in Figure 2.

(a) Find the tension in the string.

(4)

Given that the rod is about to slip,

(b) find the coefficient of friction between the rod and the ground.

(7)

Test Your Understanding *(EdExcel M2 Jan 2013 Q3)*

A ladder, of length 5 m and mass 18 kg, has one end A resting on rough horizontal ground and its other end B resting against a smooth vertical wall. The ladder lies in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan \alpha = \frac{4}{3}$, as shown in Figure 1. The coefficient of friction between the ladder and the ground is μ. A woman of mass 60 kg stands on the ladder at the point C, where $AC = 3$ m. The ladder is on the point of slipping. The ladder is modelled as a uniform rod and the woman as a particle.

Find the value of μ.

(9)

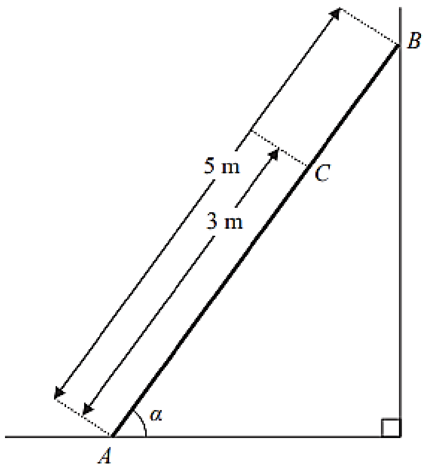


Figure 1

Exercise 7D Page 144

5. Dynamics and Inclined Planes

If a particle is in motion, $F = F_{max} = \mu R$, and F opposes the direction of motion.

- Draw a force diagram
- Use Newton's 2nd law to resolve parallel and perpendicular to the plane
- Use SUVAT to solve problems if F (and therefore a) are constant

Example *(EdExcel M1 Jan 2010 Q5)*

A particle of mass 0.8 kg is held at rest on a rough plane. The plane is inclined at 30° to the horizontal. The particle is released from rest and slides down a line of greatest slope of the plane. The particle moves 2.7 m during the first 3 seconds of its motion. Find

(a) the acceleration of the particle,

(3)

(b) the coefficient of friction between the particle and the plane.

(5)

The particle is now held on the same rough plane by a horizontal force of magnitude X newtons, acting in a plane containing a line of greatest slope of the plane, as shown in Figure 3. The particle is in equilibrium and on the point of moving up the plane.

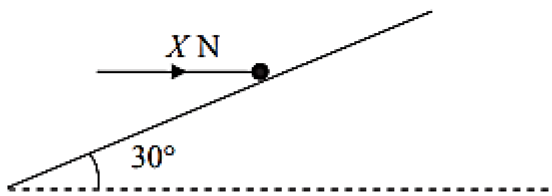


Figure 3

(c) Find the value of X.

(7)

Test Your Understanding *(EdExcel M1 May 2013(R) Q5)*

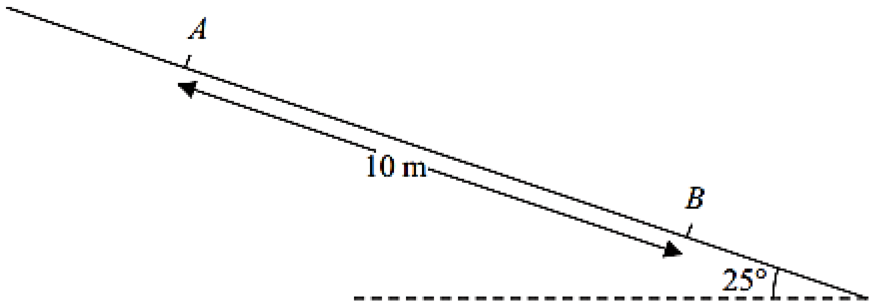


Figure 3

A particle P of mass 0.6 kg slides with constant acceleration down a line of greatest slope of a rough plane, which is inclined at 25° to the horizontal. The particle passes through two points A and B, where $AB = 10$ m, as shown in Figure 3. The speed of P at A is 2 m s^{-1}. The particle P takes 3.5 s to move from A to B. Find

(a) the speed of P at B,

(3)

(b) the acceleration of P,

(2)

(c) the coefficient of friction between P and the plane.

(5)

Exercise 7E Page 148

6. Connected Particles

Use Newton's 2nd law, SUVAT and $F_{max} = \mu R$ to solve problems about connected particles on rough and inclined surfaces.

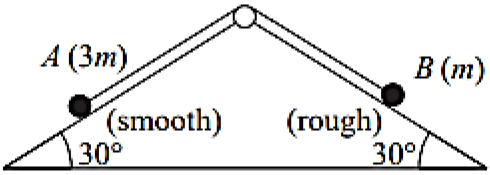
Example

Two particles P and Q, of mass 2kg and 3kg respectively, are connected by a light, inextensible string. The string passes over a small smooth pulley which is fixed at the top of a rough inclined plane. The plane is inclined to the horizontal at an angle of 30^0. Particle P is held at rest on the inclined plane and Q hangs freely on the edge of the plane with the string vertical and taut. Particle P is released and it accelerates up the plane at $2.5ms^{-2}$. Find:

a) The tension in the string
b) The coefficient of friction between P and the plane
c) The force exerted by the string on the pulley

Example *(EdExcel M1 Jan 2006 Q7)*

Figure 3



A fixed wedge has two plane faces, each inclined at 30° to the horizontal. Two particles A and B, of mass $3m$ and m respectively, are attached to the ends of a light inextensible string. Each particle moves on one of the plane faces of the wedge. The string passes over a small smooth light pulley fixed at the top of the wedge. The face on which A moves is smooth. The face on which B moves is rough. The coefficient of friction between B and this face is μ. Particle A is held at rest with the string taut. The string lies in the same vertical plane as lines of greatest slope on each plane face of the wedge, as shown in Figure 3.

The particles are released from rest and start to move. Particle A moves downwards and B moves upwards. The accelerations of A and B each have magnitude $\frac{1}{10}g$.

(a) By considering the motion of A, find, in terms of m and g, the tension in the string. **(3)**

(b) By considering the motion of B, find the value of μ. **(8)**

(c) Find the resultant force exerted by the string on the pulley, giving its magnitude and direction. **(3)**

Additional Question (Connected Particles)

EdExcel M1 (Old) Jan 2013 Q7

7.

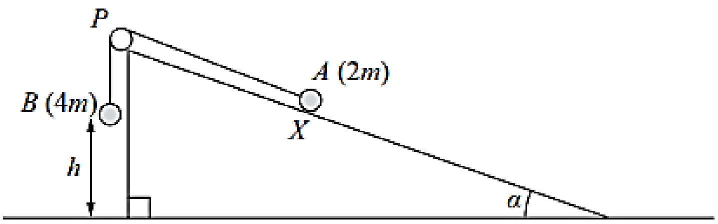


Figure 5

Figure 5 shows two particles A and B, of mass $2m$ and $4m$ respectively, connected by a light inextensible string. Initially A is held at rest on a rough inclined plane which is fixed to horizontal ground. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$. The coefficient of friction between A and the plane is $\frac{1}{4}$. The string passes over a small smooth pulley P which is fixed at the top of the plane. The part of the string from A to P is parallel to a line of greatest slope of the plane and B hangs vertically below P. The system is released from rest with the string taut, with A at the point X and with B at a height h above the ground.

For the motion until B hits the ground,

(a) give a reason why the magnitudes of the accelerations of the two particles are the same,

(1)

(b) write down an equation of motion for each particle,

(4)

(c) find the acceleration of each particle.

(5)

Particle B does not rebound when it hits the ground and A continues moving up the plane towards P. Given that A comes to rest at the point Y, without reaching P,

(d) find the distance XY in terms of h.

(6)

Test Your Understanding *(EdExcel M1 May 2013(R) Q3)*

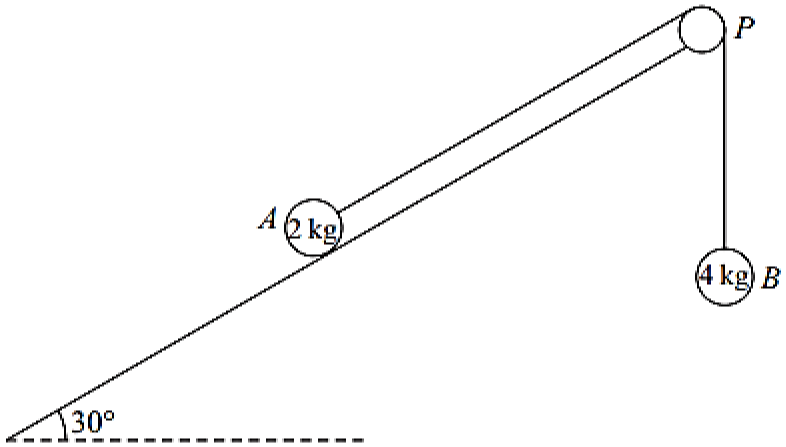


Figure 2

A fixed rough plane is inclined at 30° to the horizontal. A small smooth pulley P is fixed at the top of the plane. Two particles A and B, of mass 2 kg and 4 kg respectively, are attached to the ends of a light inextensible string which passes over the pulley P. The part of the string from A to P is parallel to a line of greatest slope of the plane and B hangs freely below P, as shown in Figure 2. The coefficient of friction between A and the plane is $\frac{1}{\sqrt{3}}$. Initially A is held at rest on the plane. The particles are released from rest with the string taut and A moves up the plane.

Find the tension in the string immediately after the particles are released.

(9)

Exercise 7F Page 152
Mixed Exercise 7 Page 154