A Level Mathematics

Chapter 6 - Mechanics

Projectiles

Chapter Overview

1. Horizontal Projection

2. Horizontal and Vertical Components

3. Projection at any Angle

4. Projectile Motion Formulae

A particle moving in a vertical plane under gravity is sometimes called a projectile. You can use projectile motion to model the flight of e.g. a golf ball.

1. **Horizontal Motion**

The horizontal motion of a projectile is modelled as having constant velocity ($a = 0$), so $s = vt$. Use$u\_{x}$and$v\_{x}$to denote horizontal velocity components.

The vertical motion of a projectile is modelled as having constant acceleration due to gravity ($a = g$). Use SUVAT - careful with directions! Use $u\_{y}$and $v\_{y}$ to denote vertical velocity components.

**Example**

A ball is thrown horizontally with speed 20ms-1, from the top of a building, which is 30m high. Find:

a) The time the ball takes to reach the ground.

b) The distance between the bottom of the building and the point where the ball hits the ground.

**Example**

A particle is projected horizontally with a velocity of 39.2ms-1. Find the horizontal and vertical components of the velocity of the particle 3s after projection. Find also the speed and direction of the motion of the particle.

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1. **Horizontal and Vertical Components of Velocity**

When a particle is projected with initial velocity U at an angle  above the horizontal:

* The horizontal component of the initial velocity is Ucos
* The vertical component of the initial velocity is Usin
* When the particle is at its highest point, the vertical velocity = 0.
* The speed of the object is the magnitude of the velocity vector.



**Example** *(Textbook Exercise 6B Q4)*

A particle is projected from the top of a building with initial velocity of 28ms-1 at an angle ** below the horizontal, where $\tan(θ)=\frac{7}{24}$.

a) Find the horizontal and vertical components of the initial velocity

b) Express the initial velocity as a vector in terms of ***i***and ***j***.

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1. **Projection at Any Angle**

We can solve problems with particles projected at any angle by resolving the initial velocity into horizontal and vertical components.

**Range** = distance from point at which the particle was projected to the point where it strikes the horizontal plane

**Time of Flight** = time taken by particle to move from its point of projection to the point where it strikes the horizontal plane

A projectile reaches its point of greatest height when the vertical component of its velocity, $u\_{y} = 0$.

**Example**

A particle is projected from a point on a horizontal plane and has an initial velocity of $28\sqrt{3}ms^{-1}$ at an angle of elevation of 60O. Find the greatest height reached by the particle and the time taken to reach this point. Also find the range of the particle.

**Example**

A golfer hits a ball with a velocity of 52ms-1, at an angle  above the horizontal where $\tan(α) =\frac{5}{12}.$

a) Set up a mathematical model, stating any assumptions made

b) Determine the time for which the ball is at least 15m above the ground (take g = 10ms-2)

**Test Your Understanding *(EdExcel M2 May 2012 Q7)***



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**Extension Question:**

A ball is projected from ground level at an angle of $θ$. Prove that when the ball hits the ground, the distance the ball has travelled along the ground is maximised when $θ=45°$.

(Year 2 differentiation knowledge required)

1. **Projection motion Formulae**

You must be able to derive general formulae related to the motion of a particle which is projected from a point on a horizontal plane and moves freely under gravity.

**Deriving the Time of Flight (T) and the Range (R)**



A particle is projected from a point on a horizontal plane with an initial velocity $U$ at an angle $α$ above the horizontal and moves freely under gravity until it hits the plane at point $B$.

Given that that acceleration due to gravity is $g$, find expressions for:

1. the time of flight, $T$
2. the range, $R$, on the horizontal plane.

**Deriving the Equation of the Trajectory**

When a particle is projected from a point O, on a horizontal plane, the equation of the trajectory may be obtained by taking x and y axes through the point of projection, O, as shown on the diagram.



A particle is projected from a point with speed $U$ at an angle of elevation $α$ and moves freely under gravity. When the particle has moved a horizontal distance $x$, its height above the point of projection is $y$.

1. Show that $y=x\tan(α -\frac{gx^{2}}{2u^{2}}\left(1+tan^{2}α\right))$

A particle is projected from a point $O$ on a horizontal plane, with speed 28 ms-1 at an angle of elevation $α$. The particle passes through a point $B$, which is at a horizontal distance of 32m from $O$ and at a height of 8m above the plane.

(b) Find the two possible values of $α$, giving your answers to the nearest degree.

**Exam Note**: You may be asked to derive these. But don’t attempt to memorise them or actually use them to solve exam problems – instead use the techniques used earlier in the chapter.

For a particle projected with initial velocity $U$ at angle $α$ above horizontal and moving freely under gravity:

* Time of flight $=\frac{2U\sin(α)}{g}$
* Time to reach greatest height $=\frac{U\sin(α)}{g}$
* Range on horizontal plane $=\frac{U^{2}\sin(2α)}{g}$
* Equation of trajectory: $y=x\tan(α -\frac{gx^{2}}{2U^{2}}\left(1+tan^{2}α\right))$
where $y$ is vertical height of particle and $x$ horizontal distance.

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