Chapter 4 - Mechanics

Moments

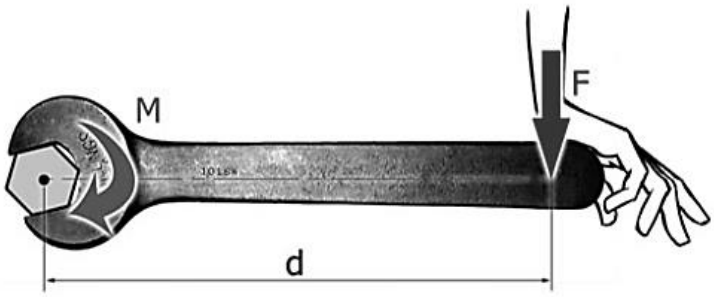
Chapter Overview

1. Moments
2. Resultant Moments
3. Equilibrium
4. Centres of Mass
5. Tilting

Topics	What students need to learn:		
	Content		Guidance
9 **Moments**	9.1	Understand and use moments in simple static contexts.	Equilibrium of rigid bodies. Problems involving parallel and non-parallel coplanar forces, e.g. ladder problems.

1. Moments

The moment of a force is the **turning effect** of the force on the body on which it is acting.



The moment is dependent on:

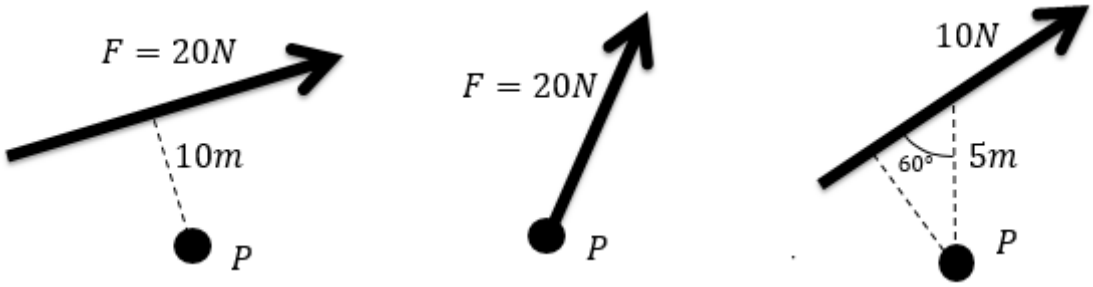
- The **magnitude** of the force
- The **distance** of the force from the axis of rotation

Moment of force =

You must also give the **direction** of the force.

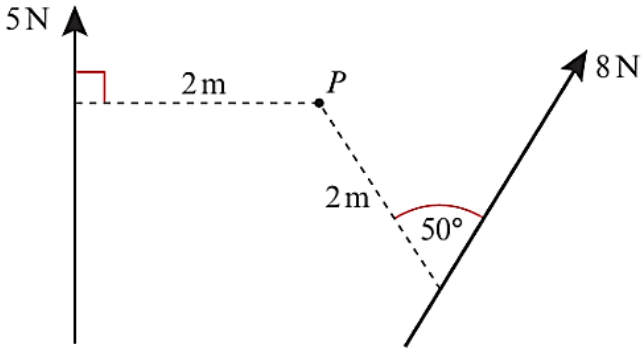
Example

In each diagram, find the moment of the force, F, about the point P.



Test Your Understanding *(Textbook))*

The diagram shows two forces acting on a lamina. Find the moment of each of the forces about P.



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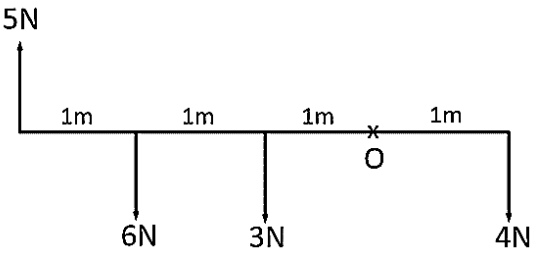
2. Resultant Moments

If several **coplanar** forces act on a body, you can add the moments about a point. Choose a positive direction (clockwise or anti-clockwise) and consider the sense of rotation of each moment.

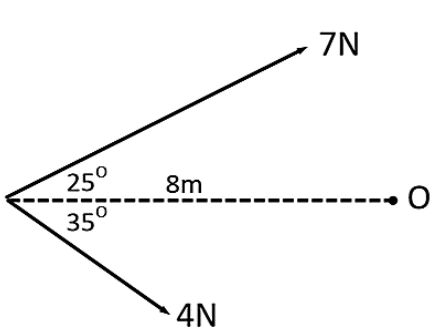
Example

Find the sum of moments about the point O, of the forces acting in each diagram:

a)



b)

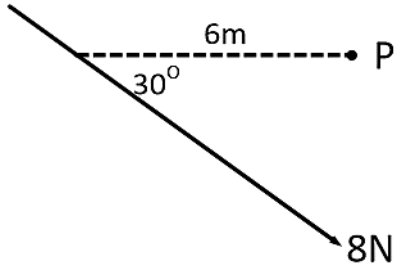


Resolving a Force into its Parallel and Perpendicular Components to Find its Moment

This can be a useful technique for harder problems. We can use trig to find the perpendicular distance of a force from a point of rotation, but we can also split the force into its parallel and perpendicular components.

Example

Find the moment of the force about point P.



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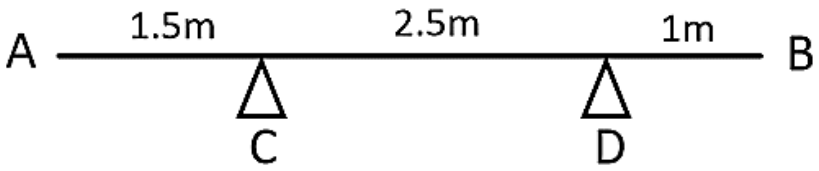
3. Equilibrium

If a rigid body is in equilibrium:

1.
2.

Example

AB is a uniform rod of length 5m and weight 20N. AB is resting in a horizontal position on supports at C and D. Find the magnitude of the reactions at C and D.



Example

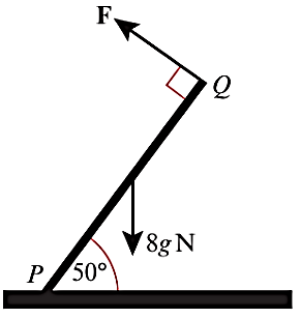
A uniform beam AB, of length 2m and mass 4kg, has a mass of 3kg attached to one end and a mass of 1kg attached at the other end. Find the position of the support C, if the beam rests in a horizontal position.

Remember to include all forces on your force diagram.
There are two ways to solve this problem:

1) Take moments about C
2) Resolve forces to find the reaction at C, then take moments about one end

Test Your Understanding *(Textbook)*

A uniform rod PQ is hinged at the point P, and is held in equilibrium at an angle of $50°$ to the horizontal by a force of magnitude $\mathbf{F}$ acting perpendicular to the rod at Q. Given that the rod has a length of 3 m and a mass of 8 kg, find the value of $\mathbf{F}$.



Test Your Understanding *(EdExcel M1 May 2013 (R) Q8)*

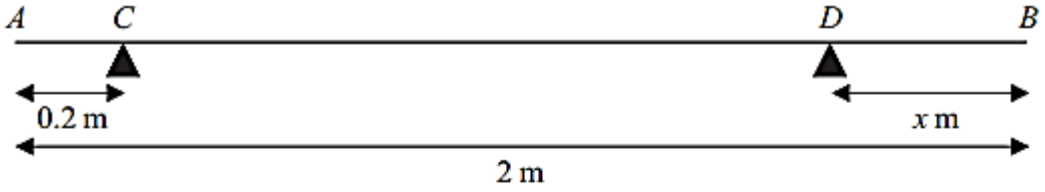


Figure 5

A uniform rod AB has length 2 m and mass 50 kg. The rod is in equilibrium in a horizontal position, resting on two smooth supports at C and D, where $AC = 0.2$ metres and $DB = x$ metres, as shown in Figure 5. Given that the magnitude of the reaction on the rod at D is twice the magnitude of the reaction on the rod at C,

(a) find the value of x.

(6)

The support at D is now moved to the point E on the rod, where $EB = 0.4$ metres. A particle of mass m kg is placed on the rod at B, and the rod remains in equilibrium in a horizontal position. Given that the magnitude of the reaction on the rod at E is four times the magnitude of the reaction on the rod at C,

(b) find the value of m.

(7)

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4. Centres of Mass (Non-Uniform Bodies)

The mass of a non-uniform rigid body can be modelled as acting at its centre of mass.

Example

Sam and Tasmin are sitting on a non-uniform plank AB, of mass 28kg and length 5m. The plank is pivoted at M, the midpoint of AB. The centre of mass of AB is at C, where AC is 2.2m. Sam has mass 40kg. Tasmin has mass 35kg and sits at A. Where must Sam sit for the plank to be horizontal?

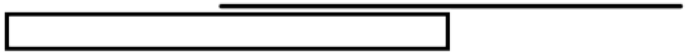
Make sure that you put all the forces on the diagram.

Example

Two sand bags of masses 7kg and 3kg are placed on the ends of a non-uniform rod PQ, of mass 8kg and length 4m, with the 7kg mass placed at P.

The rod rests in equilibrium on the edge of a smooth table, with half of the rod lying on the table's surface. Find the distance of the centre of mass from the edge of the table and the reaction force of the table on the rod.

If the rod were to tilt, where would it pivot? This will be where the reaction of the table on the rod is positioned.



Test Your Understanding *(EdExcel M1 May 2012 Q2)*

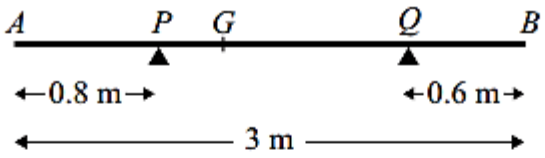


Figure 1

A non-uniform rod AB has length 3 m and mass 4.5 kg. The rod rests in equilibrium, in a horizontal position, on two smooth supports at P and at Q, where $AP = 0.8$ m and $QB = 0.6$ m, as shown in Figure 1. The centre of mass of the rod is at G. Given that the magnitude of the reaction of the support at P on the rod is twice the magnitude of the reaction of the support at Q on the rod, find

(a) the magnitude of the reaction of the support at Q on the rod,

(3)

(b) the distance AG.

(4)

Exercise 4D Page 81

5. Tilting

When a rigid body is on the point of tilting about a pivot, the reaction at any other support (or tension in any other wire/string) is zero.

Example

A uniform beam AB, of mass 12kg and length 6m rests on two pivots at P and Q, where AP = 1m and QB = 1.5m.
A particle of *M* kg is placed at A and the beam is about to tilt about the pivot at P. Find the mass of the particle and the reaction force at P.

Test Your Understanding – Suspended System *(Textbook)*

A non-uniform rod AB, of length 10 m and weight 40 N, is suspended from a pair of light cables attached to C and D where $AC = 3$ m and $BD = 2$ m.
When a weight of 25 N is hung from A the rod is on the point of rotating.
Find the distance of the centre of mass of the rod from A.

Test Your Understanding *(EdExcel M1 May 2013 Q6)*

A beam AB has length 15 m. The beam rests horizontally in equilibrium on two smooth supports at the points P and Q, where $AP = 2$ m and $QB = 3$ m. When a child of mass 50 kg stands on the beam at A, the beam remains in equilibrium and is on the point of tilting about P. When the same child of mass 50 kg stands on the beam at B, the beam remains in equilibrium and is on the point of tilting about Q. The child is modelled as a particle and the beam is modelled as a non-uniform rod.

(a) (i) Find the mass of the beam.

(ii) Find the distance of the centre of mass of the beam from A.

(8)

When the child stands at the point X on the beam, it remains horizontal and in equilibrium. Given that the reactions at the two supports are equal in magnitude,

(b) find AX.

(6)

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Mixed Exercise 4 Page 85