Lower 6 Chapter 13

Integration

Course Structure

1. Find given
2. Evaluate definite integrals, and hence the area under a curve.
3. Find areas bound between two different lines.





Integrating terms

Integration is the **opposite of differentiation.**

Consider:

If , what could ?

Examples

Find when:

 4.

 6.

7.

Test Your Understanding

Find when:

Ex 13A pg 289

Integration Notation



Examples

1. Find 2. Find

3. Find where and are constants.

Test Your Understanding



Ex 13B pg 291

Finding the Constant of Integration

Recall that when we integrate, we get a constant of integration, which could be any real value. This means **we don’t know what the exact original function was**.



Example

The curve with equation passes through . Given that , find the equation of the curve.

Test Your Understanding



Ex 13C pg 294

Definite Integral

The most useful use of integration is that **it finds the area under a graph**. Before we do this, we need to understand how to find a **definite integral**.

Examples

1.
2.
3. Given that is a constant and , show that there are two possible values for and find these values.

Extension

1. *[MAT 2009 1A]* The smallest value of

as varies, is what?

1. *[MAT 2015 1D]* Let

 and

Let . Which of the following statements are true?

1. is always bigger than
2. is always bigger than
3. They are always equal.
4. is bigger if , and is bigger if .
5. is bigger if , and is bigger if .

Ex 13D pg 297

Areas Under Curves

Consider our previous example . This definite integral gives the area bounded by the curve and the lines x = 1 and x = 5.

The definite integral gives the **area** between a positive curve , the **-axis**, and the lines and .

Example

Find the area of the finite region between the curve with equation and the -axis.

Test Your Understanding



Extension

*[MAT 2007 1H]* Given a function , you are told that

It follows that equals what?

*[MAT 2011 1G]*

A graph of the function is sketched on the axes below:

What is the value of ?

Ex 13E pg 299

Negative Areas

Sketch the curve .

Now calculate .

 Why is this result surprising?

Integration is just the sum of areas of infinitely thin rectangles, where the current value (i.e. ) is each height, and the widths are .

i.e. The area of each is

The problem is, when is negative, then is negative, i.e. a negative area!

The result is that the ‘positive area’ from 0 to 1 is cancelled out by the ‘negative area’ from 1 to 2, giving an overall ‘area’ of 0.



So how do we resolve this?

This explains the in the , which effectively means “the sum of the areas of strips, each of area . So the is not just part of the notation, it’s behaving as a physical quantity! (i.e. length

Example

Find the total area bound between the curve and the -axis.



Test Your Understanding



Extension

*[MAT 2010 1I]* For a positive number , let

Then when is what value?

*[STEP I 2014 Q3]*

The numbers and , where , are such that

1. In the case and , find the value of .
2. In the case , show that satisfies
Show further, with the help of a sketch, that there is only one (real) value of that satisfies the equation and that it lies between 2 and 3.
3. Show that , where and , and express in terms of . Deduce that

Ex 13F pg 301

Areas Between Curves and Lines

We are often interested in areas formed between curves and lines. It is important to sketch the graph to consider which areas we need to calculate.

Example

Determine the area between the lines with equations and



Example

The diagram shows a sketch of the curve with equation and the line with equation .

Find the area of the shaded region .

Test Your Understanding



If the top curve has equation and the bottom curve , the area between them is:

This means you can integrate a single expression to get the final area,

**Alternative Method:**

If the top curve has equation and the bottom curve , the area between them is:

This means you can integrate a single expression to get the final area, without any adjustment required after.

Extension

*[MAT 2005 1A]* What is the area of the region bounded by the curves and ?

*[MAT 2016 1H]* Consider two functions

For precisely which values of is the area of the region bounded by the -axis and the curve bigger than the area of the region bounded by the -axis and the curve ?

*(Your answer should be an inequality in terms of )*

Ex 13G pg 304