Lower 6 Chapter 11

Vectors

Chapter Overview

1. Add/scale factors and show vectors are parallel.
2. Calculate magnitude and direction of a vector.
3. Understand and use position vectors.
4. Solve geometric problems.
5. Understand speed vs velocity.



Vector basics

Whereas a **coordinate** represents a **position** in space, a **vector** represents a **displacement** in space.

* A vector has 2 properties:
* Direction
* Magnitude (i.e. length)

If $P$ and $Q$ are points then $\vec{PQ}$ is the vector between them.

* If two vectors $\vec{PQ}$ and $\vec{RS}$ have the same magnitude and direction, **they’re the same vector** and are **parallel**.
* $\vec{AB}=-\vec{BA} $ and the two vectors are parallel, equal in magnitude but in **opposite directions**.
* Triangle Law for vector addition:

$$\vec{AB}+\vec{BC}=\vec{AC}$$

The vector of multiple vectors is known as the **resultant vector.** (you will encounter this term in Mechanics)

* Vector **subtraction** is defined using vector addition and negation:

$$a-b=a+\left(-b\right)$$

* The zero vector $0$ (a bold 0), represents no movement.

$$\vec{PQ}+\vec{QP}=0$$

In 2D**:** $0=\left(\begin{matrix}0\\0\end{matrix}\right)$

* A **scalar** is a normal number, which can be used to ‘scale’ a vector.
* The **direction** will be the **same**.
* But the **magnitude** will be **different** (unless the scalar is 1).
* Any vector parallel to the vector $a$ can be written as $λa$, where $λ$ is a scalar.

The implication is that if we can write one vector **as a multiple of** another, then we can show they are parallel.

Example



Test your understanding



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Representing vectors



A **unit vector** is a vector of magnitude 1.

 $i$ and $j$ are unit vectors in the $x$-axis and $y$-axis respectively. We can write all vectors in terms of $i$and$j$**.**

Example

If $a=3i,     b=i+j,    c=i-2j$then**:**

1. Write$a$in vector form.
2. Find $b+2c$in$i,j$form.

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Magnitude of a vector

The magnitude $|a|$ of a vector $a$ is its length.

If $aIf a=\left(\begin{matrix}x\\y\end{matrix}\right)   \left|a\right|=\sqrt{x^{2}+y^{2}}$

Examples:



$2. a=\left(\begin{matrix}4\\-1\end{matrix}\right)$ $3. b=\left(\begin{matrix}2\\0\end{matrix}\right)$

Direction of a Vector

The direction of a vector can be found using basic trigonometry.

Examples

1. Find the angle that vector$ a=\left(\begin{matrix}4\\-1\end{matrix}\right)$ makes with the positive x axis.

2. Find the angle that vector$ b=\left(\begin{matrix}-5\\-12\end{matrix}\right)$ makes with ***j***.

Unit vector

A unit vector is a vector whose magnitude is 1.

If $a$ is a vector, then the unit vector $\hat{a} $in the same direction is

$$\hat{a}=\frac{a}{\left|a\right|}$$

Example:

Find a unit vector in the direction of $a=\left(\begin{matrix}3\\4\end{matrix}\right)$

**Test Your Understanding:** Convert the following vectors to unit vectors.

$a=\left(\begin{matrix}12\\-5\end{matrix}\right)$ $b=\left(\begin{matrix}1\\1\end{matrix}\right)$

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Position vectors

A vector used to represent a position is unsurprisingly known as a **position vector**.

A position can be thought of as a translation from the origin.

The position vector of a point $A$ is the vector$ \vec{OA}$, where $O$ is the origin. $\vec{OA}$ is usually written as $a$.

Examples

1. The points $A$ and $B$ have coordinates $\left(3,4\right)$ and $(11,2)$ respectively.

Find, in terms of $i$ and $j$:

1. The position vector of $A$
2. The position vector of $B$
3. The vector $\vec{AB}$

2. $\vec{OA}=5i-2j$ and $\vec{AB}=3i+4j$. Find:

a) The position vector of $B$.

b) The exact value of $\left|\vec{OB}\right|$ in simplified surd form.

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Solving Geometric Problems





Test your understanding



Area of a triangle example

If $\vec{AB}=3i-2j$ and $\vec{AC}=i-5j$. Determine $∠BAC$.

Extension

*[STEP 2010 Q7]*

Relative to a fixed origin $O$, the points $A$ and $B$ have position vectors $a$ and $b$, respectively. (The points $O, A$ and $B$ are not collinear.) The point $C$ has position vector $c$ given by

$$c=αa+βb,$$

where $α$ and $β$ are positive constants with $α+β<1$. The lines $OA$ and $BC$ meet at the point $P$ with position vector $p$ and the lines $OB$ and $AC$ meet at the point $Q$ with position vector $q$. Show that

$$p=\frac{αa}{1-β}$$

and write down $q$ in terms of $α, β$ and $b$.

Show further that the point $R$ with position vector $r$ given by

$$r=\frac{αa+βb}{α+β},$$

lies on the lines $OC$ and $AB$.

The lines $OB$ and $PR$ intersect at the point $S$. Prove that $\frac{OQ}{BQ}=\frac{OS}{BS}$.

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Modelling with vectors



Examples

1. A girl walks 2 km due east from a fixed point $O$ to $A$, and then 3 km due south from $A$ to $B$. Find
2. the total distance travelled
3. the position vector of $B$ relative to $O$
4. $\left|\vec{OB}\right|$
5. The bearing of $B$ from $O$.
6. In an orienteering exercise, a cadet leaves the starting point $O$ and walks 15 km on a bearing of $120°$ to reach $A$, the first checkpoint. From $A$ he walks 9 km on a bearing of $240°$ to the second checkpoint, at $B$. From $B$ he returns directly to $O$.

 Find:

1. the position vector of $A$ relative to $O$
2. $\left|\vec{OB}\right|$
3. the bearing of $B$ from $O$
4. the position vector of $B$ relative $O$.

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