9J Chain Rule in Context

1. Given that the area of a circle $A\ cm^2$ is related to its radius $r\ cm$ by the formula $A = \pi r^2$, and that the rate of change of its radius in $cm\ s^{-1}$ is given by $\frac{dr}{dt} = 5$, find $\frac{dA}{dt}$ when $r = 3$

2. The volume of a hemisphere $V\ cm^3$ is related to its radius $r\ cm$ by the formula $V = \frac{2}{3}\pi r^3$, and the total surface area $S\ cm^2$ is given by the formula $S = 3\pi r^2$. Given that the rate of increase of volume, in $cm^3\ s^{-1}$, $\frac{dV}{dt} = 6$, find the rate of increase of the surface area, $\frac{dS}{dt}$.

3. In the decay of radioactive particles, the rate at which particles decay is proportional to the number of particles remaining. Write down a differential equation for the rate of change of the number of particles.

4. Newton's law of cooling states that the rate of loss of temperature of a body is proportional to the excess temperature of the body compared to its surroundings. Write an equation that expresses this law.

5. The head of a snowman of radius $R\ cm$ loses volume by evaporation at a rate proportional to its surface area. Assuming that the head is spherical, that the volume of a sphere is given by $V = \frac{4}{3}\pi R^3\ cm^3$ and that the surface area is $A = 4\pi R^2\ cm^2$, write down a differential equation for the rate of change of radius of the snowman's head.