Relating Rates of Change

Eg. Determine the rate of change of the area A of a circle when the radius $r = 3$cm, given that the radius is changing at a rate of $5\ cm\ s^{-1}$

Firstly, how would we represent…

Fro Tip: Whenever you see the word '<u>rate</u>', think $/dt$

"the rate of change of the area A"

"the rate of change of the radius r is 5"

"the area A of a circle"

Then by Chain Rule:

First copy top and bottom into the diagonals…

$$\frac{dA}{dt} = \frac{\quad}{dr} \times \frac{dr}{\quad}$$

…Then fill in the gaps with whatever variable you didn't use.

A differential equation is an equation that can be used to calculate a rate of change over time (essentially, what you have just been doing!)

Textbook. **In the decay of radioactive particles, the rate at which particles decay is proportional to the number of particles remaining. Write down a differential equation for the rate of change of the number of particles.**

Textbook. **Newton's law of cooling states that the rate of loss of temperature of a body is proportional to the excess temperature of the body compared to its surroundings. Write an equation that expresses this law.**

Textbook. **The head of a snowman of radius $R\ cm$ loses volume by evaporation at a rate proportional to its surface area. Assuming that the head is spherical, that the volume of a sphere is given by $V = \frac{4}{3}\pi R^3\ cm^3$ and that the surface area is $A = 4\pi R^2\ cm^2$, write down a differential equation for the rate of change of radius of the snowman's head.**

Further Example

Edexcel C4 June 2008 Q3

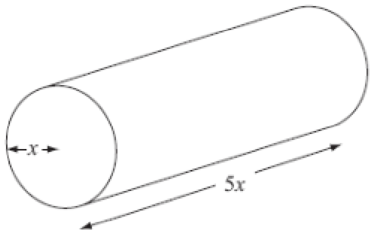

Figure 2 shows a right circular cylindrical metal rod which is expanding as it is heated. After t seconds the radius of the rod is x cm and the length of the rod is $5x$ cm.
The cross-sectional area of the rod is increasing at the constant rate of 0.032 $\text{cm}^2\,\text{s}^{-1}$.

(*a*) Find $\frac{\mathrm{d}x}{\mathrm{d}t}$ when the radius of the rod is 2 cm, giving your answer to 3 significant figures. **(4)**

(*b*) Find the rate of increase of the volume of the rod when $x = 2$. **(4)**

Test Your Understanding

June 2012 Q2

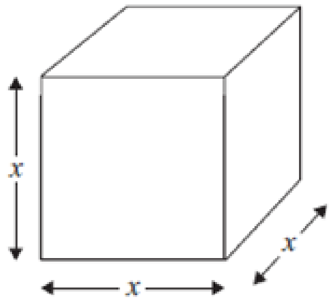

Figure 1

Figure 1 shows a metal cube which is expanding uniformly as it is heated.

At time t seconds, the length of each edge of the cube is x cm, and the volume of the cube is V cm^3.

(*a*) Show that $\frac{dV}{dx} = 3x^2$.

(1)

Given that the volume, V cm^3, increases at a constant rate of 0.048 cm^3 s^{-1},

(*b*) find $\frac{dx}{dt}$ when $x = 8$,

(2)

(*c*) find the rate of increase of the total surface area of the cube, in cm^2 s^{-1}, when $x = 8$.

(3)

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