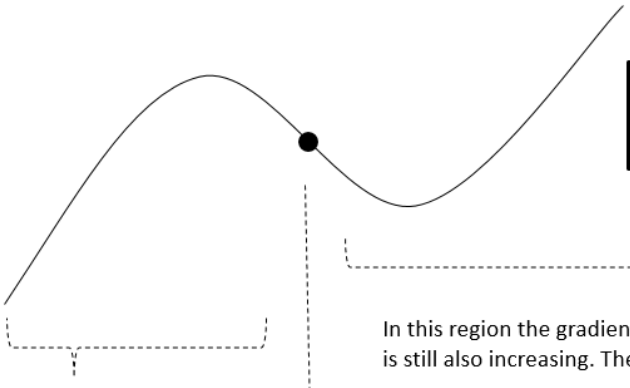
Using the second derivative

Reminder: a point of inflection is where the concavity of a curve changes, i.e. concave to convex or vice versa, or informally, 'swerving one way to swerving the other'.



Memorisation tip: Mentally visualise a positive cubic (i.e. uphill) and put 'concave' and 'convex is alphabetical order.

In this region the gradient, although initially negative, is still also increasing. The curve is **swerving left**.

We say the curve is concave when it is **swerving right**: we can see the gradient is always decreasing, i.e. the change in the gradient is negative.

At point of inflection, the gradient is not changing, i.e. the gradient of the gradient is 0. Note that points of inflection (as with this example) are not necessarily stationary points.

✎ $f(x)$ convex when $f''(x) \leq 0$

✎ $f(x)$ concave when $f''(x) \leq 0$

✎ Point of inflection when $f''(x) = 0$

Examples

1. **Find the interval on which the function $f(x) = x^3 + 4x + 3$ is concave.**

2. **Show that $f(x) = e^{2x} + x^2$ is convex for all real values of x.**

3. **The curve C has equation $y = x^3 - 2x^2 - 4x + 5$. Find the coordinates of the point of inflection.**

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