1 **a** $= 4\frac{dy}{dx}$ **b** $= 3y^2\frac{dy}{dx}$ **c** $= 2\frac{dy}{dx}\cos 2y$ **d** $= 3e^{y^2} \times 2y\frac{dy}{dx}$

$= 6ye^{y^2}\frac{dy}{dx}$

2 **a** $2x + 2y\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = -\frac{x}{y}$

b $2 - \frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$2 = \frac{dy}{dx}(1 - 2y)$

$\frac{dy}{dx} = \frac{2}{1-2y}$

c $4y^3\frac{dy}{dx} = 2x - 6$

$\frac{dy}{dx} = \frac{x-3}{2y^3}$

d $2x + 2y\frac{dy}{dx} + 3 - 4\frac{dy}{dx} = 0$

$2x + 3 = \frac{dy}{dx}(4 - 2y)$

$\frac{dy}{dx} = \frac{2x+3}{4-2y}$

e $2x - 4y\frac{dy}{dx} + 1 + 3\frac{dy}{dx} = 0$

$2x + 1 = \frac{dy}{dx}(4y - 3)$

$\frac{dy}{dx} = \frac{2x+1}{4y-3}$

f $\cos x - \frac{dy}{dx}\sin y = 0$

$\cos x = \frac{dy}{dx}\sin y$

$\frac{dy}{dx} = \frac{\cos x}{\sin y}$

g $6e^{3x} - 2e^{-2y}\frac{dy}{dx} = 0$

$6e^{3x} = 2e^{-2y}\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{3e^{3x}}{e^{-2y}} = 3e^{3x + 2y}$

h $\sec^2 x - 2\frac{dy}{dx}\operatorname{cosec} 2y \cot 2y = 0$

$\sec^2 x = 2\frac{dy}{dx}\operatorname{cosec} 2y \cot 2y$

$\frac{dy}{dx} = \frac{\sec^2 x}{2\operatorname{cosec} 2y \cot 2y}$

i $\frac{1}{x-2} = \frac{2}{2y+1}\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{2y+1}{2(x-2)}$

3 **a** $= 1 \times y + x \times \frac{dy}{dx}$

$= y + x\frac{dy}{dx}$

b $= 2x \times y^3 + x^2 \times 3y^2\frac{dy}{dx}$

$= 2xy^3 + 3x^2y^2\frac{dy}{dx}$

c $= \cos x \times \tan y + \sin x \times \frac{dy}{dx}\sec^2 y$

$= \cos x \tan y + \frac{dy}{dx}\sin x \sec^2 y$

d $= 3(x - 2y)^2 \times (1 - 2\frac{dy}{dx})$

$= 3(x - 2y)^2(1 - 2\frac{dy}{dx})$

4 **a** $2x \times y + x^2\frac{dy}{dx} = 0$

$x^2\frac{dy}{dx} = -2xy$

$\frac{dy}{dx} = -\frac{2y}{x}$

b $2x + 3 \times y + 3x \times \frac{dy}{dx} - 2y\frac{dy}{dx} = 0$

$2x + 3y = \frac{dy}{dx}(2y - 3x)$

$\frac{dy}{dx} = \frac{2x+3y}{2y-3x}$

c $8x - 2 \times y - 2x \times \frac{dy}{dx} + 6y\frac{dy}{dx} = 0$

$8x - 2y = \frac{dy}{dx}(2x - 6y)$

$\frac{dy}{dx} = \frac{4x-y}{x-3y}$

d $-2\sin 2x \times \sec 3y + \cos 2x \times 3\frac{dy}{dx}\sec 3y \tan 3y = 0$

$3\frac{dy}{dx}\cos 2x \sec 3y \tan 3y = 2\sin 2x \sec 3y$

$\frac{dy}{dx} = \frac{2\sin 2x}{3\cos 2x \tan 3y} = \frac{2}{3}\tan 2x \cot 3y$

e $\frac{dy}{dx} = 2(x + y) \times (1 + \frac{dy}{dx})$

$\frac{dy}{dx}[1 - 2(x + y)] = 2(x + y)$

$\frac{dy}{dx} = \frac{2(x+y)}{1-2(x+y)}$

f $1 \times e^y + x \times e^y\frac{dy}{dx} - \frac{dy}{dx} = 0$

$e^y = \frac{dy}{dx}(1 - xe^y)$

$\frac{dy}{dx} = \frac{e^y}{1-xe^y}$

g $2 \times y^2 + 2x \times 2y\frac{dy}{dx} - 3x^2 \times y - x^3 \times \frac{dy}{dx} = 0$

$2y^2 - 3x^2y = \frac{dy}{dx}(x^3 - 4xy)$

$\frac{dy}{dx} = \frac{2y^2-3x^2y}{x^3-4xy}$

h $2y\frac{dy}{dx} + 1 \times \ln y + x \times \frac{1}{y}\frac{dy}{dx} = 0$

$\frac{dy}{dx}(2y + \frac{x}{y}) = -\ln y$

$\frac{dy}{dx} = -\frac{\ln y}{2y+\frac{x}{y}} = -\frac{y\ln y}{2y^2+x}$

i $1 \times \sin y + x \times \frac{dy}{dx}\cos y + 2x \times \cos y + x^2 \times (-\sin y)\frac{dy}{dx} = 0$

$\sin y + 2x\cos y = \frac{dy}{dx}(x^2\sin y - x\cos y)$

$\frac{dy}{dx} = \frac{\sin y+2x\cos y}{x^2\sin y-x\cos y}$

5 **a** $2x + 2y\frac{dy}{dx} - 3\frac{dy}{dx} = 0$

$2x = \frac{dy}{dx}(3 - 2y)$

$\frac{dy}{dx} = \frac{2x}{3-2y}$

grad = 4

$\therefore\ y - 1 = 4(x - 2)$

$[\ y = 4x - 7\ ]$

b $4x - 1 \times y - x \times \frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$4x - y = \frac{dy}{dx}(x - 2y)$

$\frac{dy}{dx} = \frac{4x-y}{x-2y}$

grad = –1

$\therefore\ y - 5 = -(x - 3)$

$[\ y = 8 - x\ ]$

c $4\frac{dy}{dx}\cos y - \sec x \tan x = 0$

$4\frac{dy}{dx}\cos y = \sec x \tan x$

$\frac{dy}{dx} = \frac{\sec x \tan x}{4\cos y}$

grad $= \frac{2\times\sqrt{3}}{4\times\frac{\sqrt{3}}{2}} = 1$

$\therefore\ y - \frac{\pi}{6} = x - \frac{\pi}{3}$

$[\ y = x - \frac{\pi}{6}\ ]$

d $2\sec^2 x \times \cos y + 2\tan x \times (-\sin y)\frac{dy}{dx} = 0$

$2\sec^2 x \cos y = 2\frac{dy}{dx}\tan x \sin y$

$\frac{dy}{dx} = \frac{\sec^2 x\cos y}{\tan x \sin y}$

grad $= \frac{2\times\frac{1}{2}}{1\times\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{2}{3}\sqrt{3}$

$\therefore\ y - \frac{\pi}{3} = \frac{2}{3}\sqrt{3}\,(x - \frac{\pi}{4})$

$[\ 4\sqrt{3}\,x - 6y + \pi(2 - \sqrt{3}\,) = 0\ ]$

6 **a** $2x + 4y\frac{dy}{dx} - 1 + 4\frac{dy}{dx} = 0$

$\frac{dy}{dx}(4y + 4) = 1 - 2x$

$\frac{dy}{dx} = \frac{1-2x}{4(y+1)}$

b grad = $\frac{1}{8}$

$\therefore$ grad of normal = -8

$\therefore\ y + 3 = -8(x - 1)$

$[\ y = 5 - 8x\ ]$

7 **a** $2x + 4 \times y + 4x \times \frac{dy}{dx} - 6y\frac{dy}{dx} = 0$

$2x + 4y = \frac{dy}{dx}(6y - 4x)$

$\frac{dy}{dx} = \frac{x+2y}{3y-2x}$

grad = -4

$\therefore\ y - 2 = -4(x - 4)$

$[\ y = 18 - 4x\ ]$

b at Q, $\frac{x+2y}{3y-2x} = -4$

$x + 2y = -4(3y - 2x)$

$x = 2y$

sub. into equation of curve

$\Rightarrow\ (2y)^2 + 4y(2y) - 3y^2 = 36$

$y^2 = 4$

$y = 2$ (at P) or -2

$\therefore\ Q\ (-4, -2)$

8 $\ln y = \ln a^x$

$\ln y = x \ln a$

$\frac{1}{y}\frac{dy}{dx} = \ln a$

$\frac{dy}{dx} = y \ln a = a^x \ln a$

9 **a** $= 3^x \ln 3$

b $= 6^{2x} \ln 6 \times 2$

$= 2(6^{2x}) \ln 6$

c $= 5^{1-x} \ln 5 \times (-1)$

$= -(5^{1-x}) \ln 5$

d $= 2^{x^3} \ln 2 \times 3x^2$

$= 3x^2(2^{x^3}) \ln 2$

10 $\frac{dN}{dt} = 800(1.04)^t \times \ln 1.04$

$N = 4000$

$\therefore\ 4000 = 800(1.04)^t$

$(1.04)^t = 5$

$\frac{dN}{dt} = 800 \times 5 \times \ln 1.04 = 157$ (3sf)

$\therefore$ growing at rate of 157 per minute