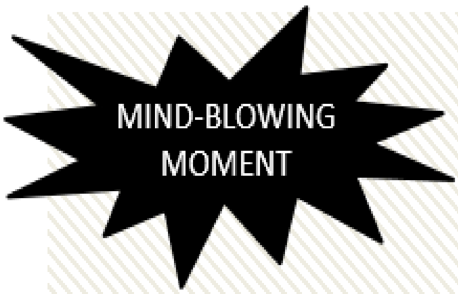
Implicit Differentiation

You're used to differentiating expressions where y is the subject, e.g. $y = x^2 + 3x$. The relationship between x and y is 'explicit' in the sense we can directly calculate y from x.

But what about **implicit** relations, e.g:

$$x^2 + y^2 = 8x \quad or \quad \cos(x+y) = \sin y$$



$$y = x^2$$

$$\frac{d}{dx} \quad \frac{dy}{dx} = 2x \quad \frac{d}{dx}$$

When seeing $y = x^2$ and differentiating, you probably think you're just differentiating the x^2. But in fact, you're differentiating **both** sides of the equation! (with respect to x)

y (by definition) differentiates to $\frac{dy}{dx}$

To differentiate implicitly you only need to know 2 things:

- Differentiate each side of the equation (using chain rule if necessary).
- Remember that y differentiated with respect to x is, by definition, $\frac{dy}{dx}$

In general, when differentiating a function of y, but with respect to x, slap a $\frac{dy}{dx}$ on the end. i.e.

$$\frac{d}{dx}\big(f(y)\big) = f'(y)\frac{dy}{dx}$$

Examples

$$\frac{d}{dx}(y^2)$$

$$\frac{d}{dx}(\sin y)$$

$$\frac{d}{dx}(e^y)$$

$$\frac{d}{dx}(xy)$$

$$\frac{d}{dx}\left(e^{x^2y}\right)$$

$$\frac{d}{dx}(\tan(x+y))$$

$$\frac{d}{dx}(x^2+\cos y)$$

Meatier Examples

[Textbook] Find $\frac{dy}{dx}$ in terms of x and y where $x^3 + x + y^3 + 3y = 6$

[Textbook] Find the value of $\frac{dy}{dx}$ at the point $(1, 1)$, where $e^{2x} \ln y = x + y - 2$

Test Your Understanding

C4 Jan 2008 Q5

A curve is described by the equation

$$x^3 - 4y^2 = 12xy.$$

(*a*) Find the coordinates of the two points on the curve where $x = -8$. **(3)**

(*b*) Find the gradient of the curve at each of these points. **(6)**

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$$x^2 + y^2 + 10x + 2y - 4xy = 10$$

(*a*) Find $\frac{dy}{dx}$ in terms of x and y, fully simplifying your answer. **(5)**

(*b*) Find the values of y for which $\frac{dy}{dx} = 0$. **(5)**

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