Parametric Differentiation

Recall from the previous chapter that parametric equations are when we define each of x and y (and possibly z) in terms of some separate parameter, e.g. t.

If x and y are given as functions of a parameter t, then

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

1. Find the gradient at the point P where $t = 2$, on the curve given parametrically by

$$x = t^3 + t, \qquad y = t^2 + 1, \ t \in \mathbb{R}$$

2. Find the equation of the normal at the point P where $\theta = \frac{\pi}{6}$, to the curve with parametric equations
$x = 3\sin\theta$, $y = 5\cos\theta$

Test Your Understanding

C4 June 2012 Q6

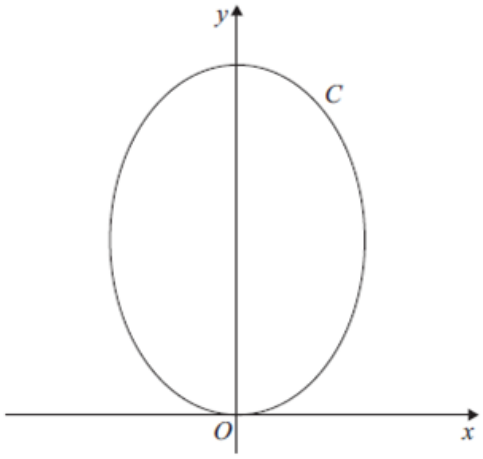

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = \sqrt{3}\sin 2t, \qquad y = 4\cos^2 t, \qquad 0 \leq t \leq \pi.$$

(*a*) Show that $\frac{dy}{dx} = k\sqrt{3}\tan 2t$, where k is a constant to be determined.

(5)

(*b*) Find an equation of the tangent to C at the point where $t = \frac{\pi}{3}$.

Give your answer in the form $y = ax + b$, where a and b are constants.

(4)