## C3 DIFFERENTIATION

## Worksheet E

1 Given that $\mathrm{f}(x)=x(x+2)^{3}$, find $\mathrm{f}^{\prime}(x)$
a by first expanding $\mathrm{f}(x)$,
b using the product rule.

2 Differentiate each of the following with respect to $x$ and simplify your answers.
a $x \mathrm{e}^{x}$
b $x(x+1)^{5}$
c $x \ln x$
d $x^{2}(x-1)^{3}$
e $x^{3} \ln 2 x$
f $x^{2} \mathrm{e}^{-x}$
g $2 x^{4}(5+x)^{3}$
h $x^{2}(x-3)^{4}$

3 Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$, simplifying your answer in each case.
a $y=x(2 x-1)^{3}$
b $y=3 x^{4} \mathrm{e}^{2 x+3}$
c $y=x \sqrt{x-1}$
d $y=x^{2} \ln (x+6)$
e $y=x(1-5 x)^{4}$
f $y=(x+2)(x-3)^{3}$
g $y=x^{\frac{4}{3}} \mathrm{e}^{3 x}$
h $y=(x+1) \ln \left(x^{2}-1\right)$
i $y=x^{2} \sqrt{3 x+1}$

4 Find the value of $\mathrm{f}^{\prime}(x)$ at the value of $x$ indicated in each case.
a $\mathrm{f}(x)=4 x \mathrm{e}^{3 x}$, $x=0$
b $\mathrm{f}(x)=2 x\left(x^{2}+2\right)^{3}$,
$x=-1$
c $\mathrm{f}(x)=(5 x-4) \ln 3 x, \quad x=\frac{1}{3}$
d $\mathrm{f}(x)=x^{\frac{1}{2}}(1-2 x)^{3}, \quad x=\frac{1}{4}$

5 Find the coordinates of any stationary points on each curve.
a $y=x \mathrm{e}^{2 x}$
b $y=x(x-4)^{3}$
c $y=x^{2}(2 x-3)^{4}$
d $y=x \sqrt{x+12}$
e $y=2+x^{2} \mathrm{e}^{-4 x}$
f $y=(1-3 x)(3-x)^{3}$

6 Find an equation for the tangent to each curve at the point on the curve with the given $x$-coordinate.
a $y=x(x-2)^{4}$,
$x=1$
b $y=3 x^{2} \mathrm{e}^{x}$,
$x=1$
c $y=(4 x-1) \ln 2 x$,
$x=\frac{1}{2}$
d $y=x^{2} \sqrt{x+6}$,
$x=-2$

7 Find an equation for the normal to each curve at the point on the curve with the given $x$-coordinate. Give your answers in the form $a x+b y+c=0$, where $a, b$ and $c$ are integers.
a $y=x^{2}(2-x)^{3}$,
$x=1$
b $y=x \ln (3 x-5)$,
$x=2$
c $y=\left(x^{2}-1\right) \mathrm{e}^{3 x}$,
$x=0$
d $y=x \sqrt{x-4}$,
$x=8$

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The diagram shows part of the curve with equation $y=x \mathrm{e}^{x^{2}}$ and the tangent to the curve at the point $P$ with $x$-coordinate 1 .
a Find an equation for the tangent to the curve at $P$.
b Show that the area of the triangle bounded by this tangent and the coordinate axes is $\frac{2}{3} \mathrm{e}$.

