DIFFERENTIATION

Worksheet E

Given that $f(x) = x(x+2)^3$, find f'(x)1

a by first expanding f(x),

b using the product rule.

Differentiate each of the following with respect to x and simplify your answers. 2

 $\mathbf{a} \quad x \mathbf{e}^x$

b $x(x+1)^5$

 $\mathbf{c} \quad x \ln x$

d $x^2(x-1)^3$

e $x^3 \ln 2x$ **f** $x^2 e^{-x}$

g $2x^4(5+x)^3$ **h** $x^2(x-3)^4$

Find $\frac{dy}{dx}$, simplifying your answer in each case. 3

a $y = x(2x-1)^3$ **b** $y = 3x^4e^{2x+3}$

 $\mathbf{c} \quad y = x\sqrt{x-1}$

 $\mathbf{d} \quad y = x^2 \ln (x+6)$

e $y = x(1 - 5x)^4$ **f** $y = (x + 2)(x - 3)^3$

 $\mathbf{g} \quad v = x^{\frac{4}{3}} e^{3x}$

h $v = (x + 1) \ln (x^2 - 1)$ **i** $v = x^2 \sqrt{3x + 1}$

Find the value of f'(x) at the value of x indicated in each case. 4

b $f(x) = 2x(x^2 + 2)^3$, x = -1

a $f(x) = 4xe^{3x}$, x = 0 **b** $f(x) = 2x(x^2 + 2)^3$, x = -1 **c** $f(x) = (5x - 4) \ln 3x$, $x = \frac{1}{3}$ **d** $f(x) = x^{\frac{1}{2}}(1 - 2x)^3$, $x = \frac{1}{4}$

5 Find the coordinates of any stationary points on each curve.

 $\mathbf{a} \quad v = x e^{2x}$

b $y = x(x-4)^3$

c $y = x^2(2x - 3)^4$

d $y = x\sqrt{x+12}$

e $y = 2 + x^2 e^{-4x}$ **f** $y = (1 - 3x)(3 - x)^3$

Find an equation for the tangent to each curve at the point on the curve with the given x-coordinate. 6

a $y = x(x-2)^4$,

x = 1 **b** $y = 3x^2 e^x$,

x = 1

c $y = (4x - 1) \ln 2x$, $x = \frac{1}{2}$ **d** $y = x^2 \sqrt{x + 6}$, x = -2

Find an equation for the normal to each curve at the point on the curve with the given *x*-coordinate. 7 Give your answers in the form ax + by + c = 0, where a, b and c are integers.

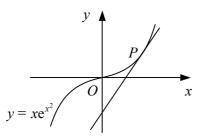
a $y = x^2(2-x)^3$,

x = 1 **b** $y = x \ln (3x - 5),$ x = 2

 $v = (x^2 - 1)e^{3x}$

x = 0 **d** $y = x\sqrt{x-4}$ x = 8

8



The diagram shows part of the curve with equation $y = xe^{x^2}$ and the tangent to the curve at the point P with x-coordinate 1.

a Find an equation for the tangent to the curve at *P*.

b Show that the area of the triangle bounded by this tangent and the coordinate axes is $\frac{2}{3}$ e.