Differentiating combinations of functions

Functions can interact in different ways…

How to differentiate

1 **Composite Function**
i.e. of form $y = f(g(x))$

$$y = \sqrt{1 + 3x}$$

The 'outer' function here is the $\sqrt{}$ and the inner function the $1 + 3x$.
i.e. $f(x) = \sqrt{x}$ and $g(x) = 1 + 3x$

→ The Chain Rule (Ex9C)

2 **Product of Two Functions**
i.e. of form $y = f(x)g(x)$

$$y = x \sin 2x$$

→ The Product Rule (Ex9D)

3 **Division (i.e. "Quotient") of Two Functions**
i.e. of form $y = \frac{f(x)}{g(x)}$

$$y = \frac{\ln x}{x}$$

→ The Quotient Rule (Ex9E)

The Chain Rule

The Chain Rule:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

The chain rule allows us to differentiate a composite function, i.e. a function within a function.

$$\text{Eg. } \boldsymbol{y = (3x^4 + x)^5}$$

Full Method:

Doing it mentally in one go:

(aka the 'bla method')

Further Practice

$y = (x^2 + 1)^3$

$y = (\ln x)^3$

$y = e^{x^2+x}$

$y = (2^x + 1)^2$

$y = \ln(\sin x)$

$y = \sin 5x$

$y = \sin^2 x =$

$y = \sqrt{x + 1} =$

$y = \cos^3 2x =$

$y = e^{e^x}$

Test Your Understanding

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Differentiate with respect to x

(*a*) $\ln(x^2 + 3x + 5)$, **(2)**

[Textbook] Given that $y = \sqrt{5x^2 + 1}$, find $\frac{dy}{dx}$ at $(4,9)$

Exercise 9C Page 239-240

Q1-5 & 11-13