U6 Chapter 9

Differentiation

Chapter Overview

1. Differentiate trigonometric, exponential and log functions.
2. Use chain, product and quotient rules.
3. Differentiate parametric equations.
4. Implicit Differentiation
5. Rates of change

7 **Differentiation**	7.1	**Understand and use the derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a general point (x, y); the gradient of the tangent as a limit; interpretation as a rate of change** **sketching the gradient function for a given curve** **second derivatives** **differentiation from first principles for small positive integer powers of** x and for $\sin x$ and $\cos x$	**Know that $\frac{dy}{dx}$ is the rate of change of y with respect to x.** **The notation $f'(x)$ may be used for the first derivative and $f''(x)$ may be used for the second derivative.** **Given for example the graph of $y = f(x)$, sketch the graph of $y = f'(x)$ using given axes and scale. This could relate speed and acceleration for example.** **For example, students should be able to use, for $n = 2$ and $n = 3$, the gradient expression** $\lim_{h \to 0}\left(\frac{(x+h)^n - x^n}{h}\right)$ **Students may use δx or h**

Topics	**What students need to learn:**		
	Content		**Guidance**
7 **Differentiation** *continued*	7.1 *cont.*	**Understand and use the second derivative as the rate of change of gradient;** connection to convex and concave sections of curves and points of inflection.	Use the condition $f''(x) > 0$ implies a minimum and $f''(x) < 0$ implies a maximum for points where $f'(x) = 0$ Know that at an inflection point $f''(x)$ changes sign. Consider cases where $f''(x) = 0$ and $f'(x) = 0$ where the point may be a minimum, a maximum or a point of inflection (e.g. $y = x^n, n > 2$)
	7.2	**Differentiate x^n, for rational values of n, and related constant multiples, sums and differences.** Differentiate e^{kx} and a^{kx}, $\sin kx$, $\cos kx$, $\tan kx$ and related sums, differences and constant multiples. Understand and use the derivative of $\ln x$	**For example, the ability to differentiate expressions such as** $(2x + 5)(x - 1)$ **and** $\frac{x^2 + 3x - 5}{4x^{\frac{1}{2}}}$, $x > 0$, **is expected.** Knowledge and use of the result $\frac{d}{dx}(a^{kx}) = ka^{kx}\ln a$ is expected.
	7.3	**Apply differentiation to find gradients, tangents and normals**	**Use of differentiation to find equations of tangents and normals at specific points on a curve.**
		maxima and minima and stationary points. points of inflection	**To include applications to curve sketching. Maxima and minima problems may be set in the context of a practical problem.**
		Identify where functions are increasing or decreasing.	**To include applications to curve sketching.**

7 **Differentiation** *continued*	7.4	Differentiate using the product rule, the quotient rule and the chain rule, including problems involving connected rates of change and inverse functions.	Differentiation of cosec x, cot x and sec x. Differentiation of functions of the form $x = \sin y$, $x = 3\tan 2y$ and the use of $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\left(\frac{\mathrm{d}x}{\mathrm{d}y}\right)}$ Use of connected rates of change in models, e.g. $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}r} \times \frac{\mathrm{d}r}{\mathrm{d}t}$ Skill will be expected in the differentiation of functions generated from standard forms using products, quotients and composition, such as $2x^4 \sin x$, $\frac{\mathrm{e}^{3x}}{x}$, $\cos^2 x$ and $\tan^2 2x$.
	7.5	Differentiate simple functions and relations defined implicitly or parametrically, for first derivative only.	The finding of equations of tangents and normals to curves given parametrically or implicitly is required.
	7.6	Construct simple differential equations in pure mathematics and in context, (contexts may include kinematics, population growth and modelling the relationship between price and demand).	Set up a differential equation using given information which may include direct proportion.

Differentiating trigonometric functions

You need to be able to differentiate sin x and cos x from first principles.

Example 1 **Prove, from first principles, that the derivative of sin x is cos x.**

$$\frac{d}{dx}(\sin x) = \mathbf{cos}\ \boldsymbol{x}$$

Things of helpfulness:

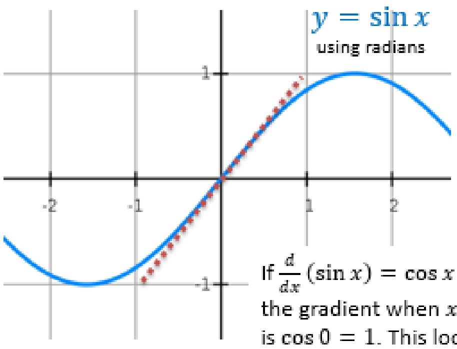
- As $x \to 0$, $\sin x \approx x$ and $\cos x \approx 1 - \frac{1}{2}x^2$
- $\sin(a+b) = \sin a \cos b + \cos a \sin b$

If $\boldsymbol{y = f(x)}$ then $\boldsymbol{\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$

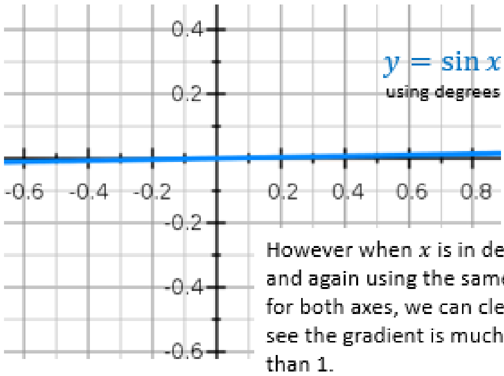
Why does this result only hold in radians?

Explanation 1:
The approximations $\sin x \approx x$ and $\cos x \approx 1 - \frac{1}{2}x^2$ (as $x \to 0$) only holds if x is in radians (we saw why in the chapter on radians). The proof that $\frac{d}{dx}(\sin x) = \cos x$ made use of these approximations.

Explanation 2:
We can see by observation if we look at the graph of $\sin x$ in radians and in degrees.



If $\frac{d}{dx}(\sin x) = \cos x$ then the gradient when $x = 0$ is $\cos 0 = 1$. This looks right from the graph.



However when x is in degrees, and again using the same scale for both axes, we can clearly see the gradient is much lower than 1.

$$\frac{d}{dx}(\sin kx) = k\cos kx$$

$$\frac{d}{dx}(\cos kx) = -k\sin kx$$

Quickfire Questions:

$\frac{d}{dx}(\sin 3x) =$

$\frac{d}{dx}(\cos 5x) =$

$\frac{d}{dx}(3\sin 5x) =$

$\frac{d}{dx}(4\cos 3x) =$

$\frac{d}{dx}\left(-\frac{1}{2}\sin x\right) =$

$\frac{d}{dx}\left(-\frac{2}{3}\cos\frac{1}{2}x\right) =$

Example

[Textbook] A curve has equation $y = \frac{1}{2}x - \cos 2x$. Find the stationary points on the curve in the interval $0 \leq x \leq \pi$.

Test Your Understanding

A curve has equation $y = \sin 3x + 2x$. Find the stationary points on the curve in the interval $0 \leq x \leq \frac{2}{3}\pi$.

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