8E Modelling

1. A plane's position at time t seconds after take-off can be modelled with the following parametric equations:

$$x = (vcos\theta)t,\ y = (vsin\theta)t,\ t > 0$$

Where v is the speed of the plane, θ is the angle of elevation of its path, x is the horizontal distance travelled (m) and y is the vertical distance travelled (m), relative to a fixed origin.

When the plane has travelled 600m horizontally, is has climbed 120m.

a) Find the angle of elevation, θ.

Given that the plane's speed is $50ms^{-1}$.

b) Find the parametric equations for the plane's motion

c) Find the vertical height of the plane after 10 seconds

d) Show that the plane's motion is a straight line

e) Explain why the domain, $t > 0$ is not realistic

2. A stone is thrown from the top of a 25m high cliff with an initial speed of $5ms^{-1}$ at an angle of 45°. Its position after t seconds can be described using the following parametric equations:

$$x = \frac{5\sqrt{2}}{2}t, \quad y = \left(-4.9t^2 + \frac{5\sqrt{2}}{2}t + 25\right)$$

$$0 \leq t \leq k$$

Where x is the horizontal distance (m), y is the vertical distance (m) from the ground, and k is a constant.

a) Given that the model is valid from the time the stone is thrown until the time it hits the ground, find the value of k.

b) Find the horizontal distance travelled by the stone by the time it hits the floor ground

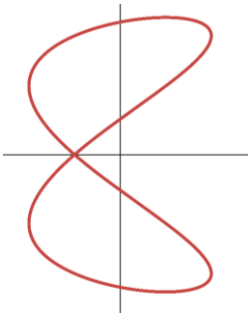
3. The motion of a figure skater relative to a fixed origin O, at time t minutes is modelled using the parametric equations:

$$x = 8cos20t, \quad y = 12sin\left(10t - \frac{\pi}{3}\right)$$

$$t \geq 0$$

Where x and y are measured in metres.

a) Find the coordinates of the figure skater at the beginning of their motion.



b) Find the coordinates of the point where the figure skater intersects their own path, given that it takes place on the x-axis

c) Find the coordinates of the points where the curve intersects the y-axis

d) Find how long it takes the figure skater to complete on figure of eight motion