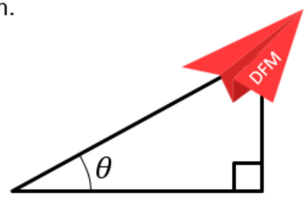
Modelling

As we saw at the start of this chapter, parametric equations are frequently used in mechanics, particularly where the (x, y) position (the Cartesian variables) depends on time t (the parameter).

[Textbook] A plane's position at time t seconds after take-off can be modelled with the following parametric equations:

$$x = (v\cos\theta)t \text{ m}, \qquad y = (v\sin\theta)t \text{ m}, \qquad t > 0$$

where v is the speed of the plane, θ is the angle of elevation of its path, x is the horizontal distance travelled and y is the vertical distance travelled, relative to a fixed origin.
When the plane has travelled 600m horizontally, it has climbed 120m.
a. find the angle of elevation, θ.
Given that the plane's speed is 50 m s^{-1},
b. find the parametric equations for the plane's motion.
c. find the vertical height of the plane after 10 seconds.
d. show that the plane's motion is a straight line.
e. explain why the domain of t, $t > 0$, is not realistic.



Further Example

[Textbook] The motion of a figure skater relative to a fixed origin, O, at time t minutes is modelled using the parametric equations

$$x = 8\cos 20t\,, \qquad y = 12\sin\left(10t - \frac{\pi}{3}\right), \qquad t \geq 0$$

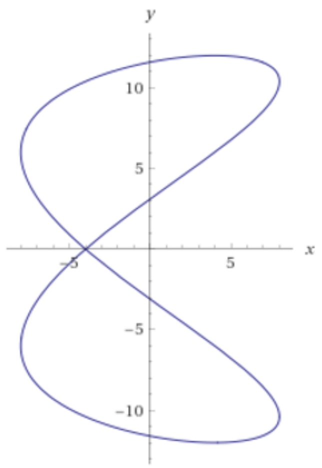
where x and y are measured in metres.

a) Find the coordinates of the figure skater at the beginning of his motion.

b) Find the coordinates of the point where the figure skater intersects his own path.

c) Find the coordinates of the points where the path of the figure skater crosses the y-axis.

d) Determine how long it takes the figure skater to complete one complete figure-of-eight motion.



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