Integrating Vectors

We can integrate vectors by integrating each function of time separately.

Remember each component will have a constant of integration, $C = (p\boldsymbol{i} + q\boldsymbol{j})$.

Example

A force $\boldsymbol{F}$ acts on a body of mass 250g which is initially at rest at a fixed point O. If $\boldsymbol{F} = ((5t - 2)\boldsymbol{i} + 4t\boldsymbol{j})$N, where t is the time for which the force has been acting on the body, find expressions for:

a) The velocity vector of the body at time t.
b) The position vector of the body at time t.

Example *(Textbook)*

A particle P is moving in a plane so that, at time t seconds, its acceleration is $(4\boldsymbol{i} - 2t\boldsymbol{j})$ms^{-2}. When $t = 3$, the velocity of P is $6\boldsymbol{i}$ ms^{-1} and the position vector of P is $(20\boldsymbol{i} + 3\boldsymbol{j})$ m with respect to a fixed origin O. Find:

(a) the angle between the direction of motion of P and $\boldsymbol{i}$ when $t = 2$

(b) the distance of P from O when $t = 0$.

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At time t seconds the velocity of a particle P is $[(4t-5)\mathbf{i}+3\mathbf{j}]\,\text{m s}^{-1}$. When $t=0$, the position vector of P is $(2\mathbf{i}+5\mathbf{j})$ m, relative to a fixed origin O.

(a) Find the value of t when the velocity of P is parallel to the vector $\mathbf{j}$.

(1)

(b) Find an expression for the position vector of P at time t seconds.

(4)

A second particle Q moves with constant velocity $(-2\mathbf{i}+c\mathbf{j})\,\text{m s}^{-1}$. When $t=0$, the position vector of Q is $(11\mathbf{i}+2\mathbf{j})$ m. The particles P and Q collide at the point with position vector $(d\mathbf{i}+14\mathbf{j})$ m.

(c) Find

(i) the value of c,

(ii) the value of d.

(5)

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