Chapter 8 - Mechanics

Further Kinematics

Chapter Overview

1. Vectors in Kinematics
2. Vector Methods with Projectiles
3. Variable Acceleration in One Dimension
4. Differentiating Vectors
5. Integrating Vectors

Topics	What students need to learn:		
	Content		Guidance
7 **Kinematics**	7.1	**Understand and use the language of kinematics: position; displacement; distance travelled; velocity; speed; acceleration.**	Students should know that distance and speed must be positive.
	7.2	**Understand, use and interpret graphs in kinematics for motion in a straight line: displacement against time and interpretation of gradient; velocity against time and interpretation of gradient and area under the graph.**	**Graphical solutions to problems may be required.**
	7.3	**Understand, use and derive the formulae for constant acceleration for motion in a straight line.** Extend to 2 dimensions using vectors.	**Derivation may use knowledge of sections 7.2 and/or 7.4** **Understand and use *suvat* formulae for constant acceleration in 2-D,** e.g. $\mathbf{v} = \mathbf{u} + \mathbf{a}t$, $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with vectors given in $\mathbf{i} - \mathbf{j}$ or column vector form. Use vectors to solve problems.

7.4	**Use calculus in kinematics for motion in a straight line:** $v = \frac{dr}{dt}$, $a = \frac{dv}{dt} = \frac{d^2r}{dt^2}$ $\mathbf{r} = \int \mathbf{v} \; dt$, $\mathbf{v} = \int \mathbf{a} \; dt$ Extend to 2 dimensions using vectors.	**The level of calculus required will be consistent with that in Sections 7 and 8 in Paper 1** and Sections 6 and 7 in Paper 2. Differentiation and integration of a vector with respect to time. e.g. Given $\mathbf{r} = t^2\mathbf{i} + t^{\frac{3}{2}}\mathbf{j}$, find $\dot{\mathbf{r}}$ and $\ddot{\mathbf{r}}$ at a given time.

1. Vectors in Kinematics

If a particle starts from the point with position vector $\boldsymbol{r}_0$, and moves with constant velocity $\boldsymbol{v}$, its displacement from its initial position at time t is given by $\boldsymbol{v}t$ and it position vector $\boldsymbol{r}$ is given by:

Example

At time $t = 0$, where t is the time (in seconds), a particle is at the point with position vector $(4\boldsymbol{i} - \boldsymbol{j})$ m and travels with velocity $(-2\boldsymbol{i} + 2\boldsymbol{j})$ ms^{-1}. Find:

a) The position vector of the particle after t seconds

b) The distance the particle is from the origin, O, after 3 seconds.

Example

A particle starts at a point 8m from O at an angle of 45° anti-clockwise from east and travels with a velocity $(-2\boldsymbol{i} - 3\boldsymbol{j})$ ms^{-1}, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors due east and north respectively.

Find the position vector of the particle after t seconds in the form $\boldsymbol{r} = \boldsymbol{r}_0 + t\boldsymbol{v}$.

Example – Using SUVAT with Vectors

A particle is initially travelling with velocity $(-2\boldsymbol{i} - 9\boldsymbol{j})$ ms^{-1} and 2 seconds later it has a velocity of $(6\boldsymbol{i} - 11\boldsymbol{j})$ ms^{-1}, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors in the directions of the positive x- and y- axes respectively. Given that the acceleration of the particle is constant, find:

a) The acceleration

b) The magnitude of the acceleration

c) The angle that the acceleration makes with the vector $\boldsymbol{j}$

Example *(Textbook p161 Example 3)*

An ice skater is skating on a large flat ice rink. At time $t = 0$ the skater is at a fixed point O and is travelling with velocity $(2.4\boldsymbol{i} - 0.6\boldsymbol{j})$ ms^{-1}.

At time $t = 20$ s the skater is travelling with velocity $(-5.6\boldsymbol{i} + 3.4\boldsymbol{j})$ ms^{-1}.

Relative to O, the skater has position vector $\boldsymbol{s}$ at time t seconds.

Modelling the ice skater as a particle with constant acceleration, find:

(a) The acceleration of the ice skater

(b) An expression for $\boldsymbol{s}$ in terms of t

(c) The time at which the skater is directly north-east of O.

A second skater travels so that she has position vector $\boldsymbol{r} = (1.1t - 6)\boldsymbol{j}$ m relative to O at time t.

(d) Show that the two skaters will meet.

<u>Test Your Understanding</u> *(EdExcel M1 May 2013(R) Q6)*

[*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively. Position vectors are given with respect to a fixed origin O.*]

A ship S is moving with constant velocity $(3\mathbf{i} + 3\mathbf{j})$ km h^{-1}. At time $t = 0$, the position vector of S is $(-4\mathbf{i} + 2\mathbf{j})$ km.

(a) Find the position vector of S at time t hours.

(2)

A ship T is moving with constant velocity $(-2\mathbf{i} + n\mathbf{j})$ km h^{-1}. At time $t = 0$, the position vector of T is $(6\mathbf{i} + \mathbf{j})$ km. The two ships meet at the point P.

(b) Find the value of n.

(5)

(c) Find the distance OP.

(4)

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